

Contract Pricing Reference Guides Volume 2

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1.0 Introduction

In this chapter, you will learn to use price index numbers to make the price adjustments necessary to analyze price and cost information collected over time.

Price Index Numbers. Price index numbers measure relative price changes from one time period to another. They are so widely used that discussions related to index numbers in contract pricing normally refers to price indexes. However, other index numbers could be used in contract pricing, particularly indexes that measure productivity.

Simple and Aggregate Price Index Numbers. Price index numbers can indicate price changes for one or several related supplies or services over a period of time. The Bureau of Labor Statistics (BLS) publishes numerous simple and aggregate Producer Price Indexes (PPI) that track changes in the wholesale price of products sold in the United States.

- Simple index numbers calculate price changes for a single item over time. Index numbers are more accurate if they are constructed using actual prices paid for a single commodity, product or service rather than the more general aggregated index. An example of a simple index would be one that tracks only lemons or oranges.
- Aggregate index numbers calculate price changes for a group of related items over time. An example of an aggregate price index would be one that tracks citrus fruits.

1.1 Identifying Situations For Use

Situations for Use. You can use price index numbers to:

- Inflate/deflate prices or costs for direct comparison. You can use price index numbers to estimate/analyze product price/cost today using the price/cost of the same or a similar product in the past.
- Inflate/deflate prices or costs to facilitate trend analysis. You can use index numbers to facilitate trend or time series analysis of prices/costs by eliminating or reducing the effects of inflation so that the analysis can be made in constant-year dollars (dollars free of changes related to inflation/deflation).
- Estimate project price or cost over the period of contract performance. Prices/costs of future performance are not certain. One effect that you must consider is the changing value of the dollar. You can use index numbers to estimate and negotiate future costs and prices.
- Adjust contract price or cost for inflation/deflation. When price/cost changes are particularly volatile, you may need to include an Economic Price Adjustment (EPA) clause in the contract. The use of index numbers is one of the most popular methods used to identify and define price changes for economic price adjustment.

1.2 Constructing Price Index Numbers

Steps in Price Index Number Development. If your activity repeatedly buys the same types of services or supplies, consider developing your own price indices to track trends in price over time. This section will demonstrate the procedures for developing a simple price index. To develop an aggregate index, follow the same basic steps using data from the various products selected for index development.

There are four steps to developing a simple price index number:

Step 1. Collect data for each period.

Step 2. Select an appropriate base period.

Step 3. Divide each period price by the base-period price.

Step 4. Multiply by 100 to produce an index number.

Example of Price Index Number Development.

Step 1. Collect Data for Each Period. For each index period, collect average price data for the product, commodity, or service. For example, assume the following average yearly prices for a hoist:

Year	20X4	20X5	20X6	20X7	20X8
Price	\$84.12	\$90.84	\$95.06	\$101.97	\$107.32

Step 2. Select an Appropriate Base Period. Select a base period appropriate for the data available. In this case, we will use the 20X4 price, \$84.12.

Select Base Period		
A	B	C
Year	Average Annual Price	20X4 Base Price
20X4	\$84.12	\$84.12
20X5	\$90.84	\$84.12
20X6	\$95.06	\$84.12
20X7	\$101.97	\$84.12
20X8	\$107.32	\$84.12

Step 3. Divide each period price by the base-period price. In this example, divide each period price (Column B) by the base-period price (Column C). The result is a price relative (Column E) as shown below. A price relative is the relationship of the price in any period to the base period price. For example, the table below shows that the price in 20X6 is 1.13 times or 13 percent higher than the price in 20X4.

Calculate Price Index				
A	B	C	D	E
	Average Annual	20X4	Price Relative Calculation	Price

Year	Price	Base Price		Relative
20X4	\$84.12	\$84.12	$\frac{\$84.12}{\$84.12}$	1.000
20X5	\$90.84	\$84.12	$\frac{\$90.84}{\$84.12}$	1.080
20X6	\$95.06	\$84.12	$\frac{\$95.06}{\$84.12}$	1.130
20X7	\$101.97	\$84.12	$\frac{\$101.97}{\$84.12}$	1.212
20X8	\$107.32	\$84.12	$\frac{\$107.32}{\$84.12}$	1.276

Step 4. Convert to an Index Number. Convert to an index number (Column F) by multiplying each price relative (Column E) by 100. Normally, you should round index numbers to the nearest tenth.

Calculate Price Index					
A	B	C	D	E	F
Year	Average Annual Price	20X4 Base Price	Price Relative Calculation	Price Relative	Index Number
20X4	\$84.12	\$84.12	$\frac{\$84.12}{\$84.12}$	1.000	100.0
20X5	\$90.84	\$84.12	$\frac{\$90.84}{\$84.12}$	1.080	108.0
20X6	\$95.06	\$84.12	$\frac{\$95.06}{\$84.12}$	1.130	113.0
20X7	\$101.97	\$84.12	$\frac{\$101.97}{\$84.12}$	1.212	121.2
20X8	\$107.32	\$84.12	$\frac{\$107.32}{\$84.12}$	1.276	127.6

1.3 Selecting A Price Index For Analysis

Points to Consider in Index Selection. Use published indexes carefully as they often do not fit the pattern of price changes for the product or service you are analyzing. The data usually represent national or regional averages in lieu of any specific contractor or location. Nevertheless, price

index numbers offer a practical alternative to the costly and time-consuming task of developing index numbers from basic cost data.

When you use published indexes, choose the index series that best fits your specific analysis effort. Usually, the closer the chosen index series relates to the item you are pricing, the more useful the number will be in your analysis.

If you are buying a finished good, indices representing raw materials and purchased components may not necessarily provide an accurate basis for projecting prices. The finished good price may also be strongly influenced by trends in direct labor, cost of capital, etc. Accuracy can be improved by the use of a weighted average index which represents changes in both labor and material elements of price. Many contracting organizations develop weighted average indexes for major products or major groups of products.

Sources of Published Indexes. You may not have the time or data required to develop the price indexes that you need for a price or cost analysis. Fortunately, there are many sources of previously constructed price indexes that you can use to estimate price changes. These sources include:

- Bureau of Labor Statistics (BLS);
- Other Government agencies;
- Government contracting organizations;
- Commercial forecasting firms;
- Industry or trade publications; and
- Newspapers.

Indexes from the Bureau of Labor Statistics. The Government collects and publishes vast amounts of data on prices. Four of the best known sources of index numbers are published by the BLS:

- **Producer Price Index Detailed Report.** Probably the best known and most frequently used source of price index numbers for material pricing is the [Producer Price Index \(PPI\) Detailed Report](#) published monthly by the [U.S. Department of Labor, Bureau of Labor Statistics \(BLS\)](#) . The indexes report monthly price changes at the producer/wholesale level for 15 major commodity groups:

Producer Price Indexes Commodity Groups	
Commodity Code	Commodity Description
01	Farm Products
02	Processed Foods and Feeds
03	Textile Products and Apparel
04	Hides, Skins, Leather, and

	Related Products
05	Fuels and Related Products and Power
06	Chemicals and Allied Products
07	Rubber and Plastic Products
08	Lumber and Wood Products
09	Pulp, Paper, and Allied Products
10	Metals and Metal Products
11	Machinery and Equipment
12	Furniture and Household Durables
13	Nonmetallic Mineral Products
14	Transportation Equipment
15	Miscellaneous Products

- Consumer Price Index Detailed Report. The consumer price index (CPI), published monthly in the [Consumer Price Index Detailed Report](#) , reports on changes in consumer prices for a fixed mix of goods selected from the following categories:
 - Food;
 - Clothing;
 - Shelter and fuels;
 - Transportation; and
 - Medical services.

You should normally not use the CPI in adjusting material prices because the CPI reflects retail rather than wholesale price changes. However, the CPI can be of value in pricing services when labor rate increases are linked to changes in the CPI.

- Monthly Labor Review. The [Monthly Labor Review](#) includes selected data from a number of Government indexes, including:
 - Employment Cost Index;
 - Consumer Price Index;
 - Producer Price Indexes;
 - Export Price Indexes; and
 - Import Price Indexes.

The data and other information presented in the publication can prove useful in analyzing prices, especially on service contracts, where direct labor is a significant part of contract price.

- Employment Cost Index. The [Employment Cost Index](#) presents information on the changes in earnings index for various classes of labor. Like the Monthly Labor Review, the report can be very useful in pricing contracts in which direct labor is a significant part of the contract price.

Indexes from Other Government Agencies. Data on contract prices are also available from agencies other than the BLS, such as the Federal Reserve System and the Bureau of Economic Analysis.

- Federal Reserve System. The Board of Governors publishes the [Federal Reserve Bulletin](#), which includes economic indexes and data on business, commodity prices, construction, labor, manufacturing, and wholesale trade. Each bank in the system publishes information each month with special reference to its own Federal Reserve District.
- Bureau of Economic Analysis Publications. The Bureau of Economic Analysis, Department of Commerce, publishes the [Survey of Current Business](#) that provides general information on trends in industry and the business outlook. It furnishes economic indexes on business, construction, manufacturing, and wholesale trade.
- Indexes from Government Contracting Organizations. Many Government contracting organizations have teams of analysts who develop indexes that are particularly applicable to the organizations' specific contracting situations. These indexes may be developed from raw price data, or they may be developed as weighted averages of published indexes.

Indexes from Commercial Forecasting Firms. Numerous commercial indexes are available for use in contract price analysis. While most Government indexes only report historical price changes, many commercial indexes also forecast future price movement. In situations where forecasts are necessary, commercial indexes may prove particularly useful. Before using such indexes, examine their development and consult with auditors, technical personnel, and other contracting professionals to assure they are applicable in your analysis situation. Many federal government departments/agencies have contracts with commercial firms that provide these economic forecasting services. Check with your department or agency to see if you have access to such a service.

Indexes from Industry or Trade Publications. Industry and trade publications frequently provide general forecasts of economic conditions and price changes anticipated in the industry. To identify which publications have economic information relevant to a particular product, ask Government technical personnel. Contractors can also assist you in the identification of appropriate publications. However, be sure to verify with Government personnel the appropriateness of any information sources recommended by a contractor.

Indexes from Newspapers. Publications, such as local, national, and financial newspapers, provide valuable forecasts of price changes in specific industries. The information reported is normally data provided by the Government, economic forecasting firms, or industry groups.

1.4 Adjusting Price/Cost For Analysis

In this section, you will learn how to use price index numbers to adjust prices and costs for analysis.

- 1.4.1 - [Adjusting Price/Cost for Pricing Comparisons](#)
- 1.4.2 - [Adjusting Price/Cost For Further Analysis](#)

Compensating for Inflation or Deflation. The changing value of the dollar can complicate comparisons and other analyses using price or cost information collected over time. You can use price indexes to adjust prices/costs to compensate for inflation or deflation to facilitate direct comparisons and further analysis.

Calculate Relative Price Change Between Two Periods. Index numbers indicate the percentage change in price relative to the base year. For example, the table below shows that the average product price increased by 23.2 percent between 20X4 and 20X9.

Year	Product Index
20X4	100.0
20X5	105.3
20X6	112.0
20X7	116.5
20X8	119.3
20X9	123.2

To adjust prices for inflation or deflation, you must be able to do more than determine how prices have changed relative to the base year. You must be able to determine how prices changed between any two time periods. For example, looking at the table above, how did prices change between 20X6 and 20X9? To calculate the percentage price change between any two time periods, you must follow the same procedure that you would follow if you had actual price data; you must divide.

$$\text{Index in 20X9} \div \text{Index in 20x6} = 123.2 \div 112.0 = 1.10$$

Based on the price index and this calculation, you could estimate that product prices in 20X9 were 1.10 times the prices in 20X6 or 10.0 percent more than the prices in 20X6.

Estimating Price/Cost Using Index Numbers. You can use index numbers to adjust prices or costs from any time period for inflation or deflation. For example, the calculation above demonstrated that product prices increased 10.0 percent between 20X6 and 20X9. If you knew that the price for an equipment item in 20X6 was \$1,000, you could estimate that the price should be 10.0 percent higher in 20X9. That would result in a price estimate of \$1,100 for 20X9.

These calculations can be formalized into a simple equation using either the Ratio Method or the Price Adjustment Formula Method described below.

- Ratio Method. The Ratio Method uses an equation in the form of a simple ratio to make the price adjustment.

$$I_2 I_1 = P_2 P_1$$

Where:

I_1 = Index in Time Period 1 -- the index for the period for which you have historical cost/price information.

I_2 = Index in Time Period 2 -- the index for the period for which you are estimating.

P_1 = Price/cost in Time Period 1 -- historical cost/price information.

P_2 = Price/cost in Time Period 2 -- cost/price estimate.

Example: You purchased an item in 20X6 for \$1,000 and you are trying to estimate the price in 20X9. The relevant index in 20X6 was 112.0. In 20X9, it is 123.2.

$$I_2 I_1 = P_2 P_1$$

$$123.2 \ 112.0 = P_2 \ \$1,000$$

$$123.2 \times \$1,000 = 112.0 \times P_2$$

$$123,200 = 112.0 \times P_2$$

$$123,200 \ 112.0 = P_2$$

$$\$1,100 = P_2$$

- Price Adjustment Formula Method. The Price Adjustment Formula is a simplification of the Ratio Method described above.

$$P_2 = I_2 I_1 \times P_1$$

Example: The calculations below use the same pricing information used above to demonstrate the ratio method.

$$\begin{aligned}
P_2 &= I_2 I_1 \times P_1 \\
&= 123.2 \ 112.0 \times \$1,000 \\
&= 1.10 \times \$1,000 \\
&= \$1,100
\end{aligned}$$

Adjustment Period Selection. When adjusting historical prices for inflation, take care in selecting the period of adjustment. There are two basic methods you can use in adjusting costs/prices:

- Adjustment based on period between acquisition dates.
 - This is the method most commonly used to calculate the period of price adjustment, because acquisition dates are readily available.

For example: An item is being acquired in January 20X2 was last purchased in January 20X1. Using this method, the logical adjustment period would be January 20X1 to January 20X2 -- a year of inflation or deflation.

- - If delivery schedules are similar, this method should be satisfactory. However, if delivery schedules are significantly different, you may be over or under the adjustment required.

For example: If the January 20X1 acquisition provided for delivery in January 20X2 and the January 20X2 acquisition also provided for delivery in January 20X2, allowing for a year of inflation or deflation would likely overestimate the adjustment required. The pricing of the first acquisition should have already considered the anticipated price changes between January 20X1 and January 20X2. Why make a second adjustment for the same price changes?

- Adjustment based on period between delivery dates.
 - This method for determining the appropriate period of adjustment is probably more accurate for the reasons described above. The problem with applying this method is the collection of accurate information on delivery dates. Application may be further complicated by phased deliveries over an extended period of time.
 - For smaller dollar material purchases in periods of limited price changes, the differences between acquisition date to acquisition data and delivery date to delivery date adjustment may not be that significant. However, as contract costs/prices increase or cost/price changes become more volatile, selection of the proper adjustment period becomes more important.
 - Wage rates should always be estimated for the time period in which the work will be performed.

1.4.1 Adjusting Price/Cost For Pricing Comparisons

Should-Pay Estimates. You can use price indexes to develop should-pay estimates of current price or cost based on historical information. These should-pay estimates can be used for a

variety of purposes including comparison with an offered price or cost as part of an evaluation of reasonableness.

Steps in Using Price Indexes to Analyze Price/Cost Reasonableness. To perform this analysis, follow the steps below:

Step 1. Collect available price/cost data.

Step 2. Select a price index for adjusting price/cost data.

Step 3. Adjust price/cost for inflation/deflation.

Step 4. Use adjusted price/cost for pricing comparisons.

Example of Using Price Indexes to Analyze Price/Cost Reasonableness. Consider the problem of analyzing a contractor's proposed price of \$23,000 for a turret lathe to be delivered in 20X8.

Step 1. Collect available price/cost data. A procurement history file reveals that the same machine tool was purchased in 20X4 at a price of \$18,500. Determine whether the 20X8 proposed price is reasonable.

Step 2. Select a price index for adjusting price/cost data. Select or construct an appropriate index. In this case, you might select a Machinery and Equipment Index as a reasonable indicator of price movement for a turret lathe. You could extract the data from a Government publication (e.g., the PPI) or use a similar commercial index.

Year	Machinery and Equipment Index
20X2	100.0
20X3	103.3
20X4	106.0
20X6	110.8
20X7	115.0
20X8	121.9

Step 3. Adjust price/cost for inflation/deflation. After you have selected an index, you can adjust prices to a common dollar value level. In this case, you would normally adjust the historical 20X4 price to the 20X8 dollar value level. To make the adjustment, you simply use one of the methods already demonstrated.

Using the Ratio Method.

$$I_2 I_1 = P_2 P_1$$

$$121.9 \ 106.0 = \text{Price Estimate for 20X8 } \$18,500$$

$$121.9 \times \$18,500 = 106.0 \times P_2$$

$$\$2,255,150 = 106.0 \times P_2$$

$$2,255,150 \ 106.0 = P_2$$

$$\$21,275 = P_2$$

Using the Formula Adjustment Method.

$$\begin{aligned} P_2 &= I_2 I_1 \\ &= 121.9 \ 106.0 \times \$18,500 \\ &= 1.15 \times \$18,500 \\ &= \$21,275 \end{aligned}$$

Step 4. Use adjusted price/cost for pricing comparisons. Once you have made the adjustment for inflation/deflation, you can compare the offered and historical prices in constant dollars. The offered price/cost is \$23,000, but the adjusted historical price/cost is only \$21,275. Thus, the offered price/cost is \$1,725, or 8.1 percent higher than what you would expect, given the historical data and available price indexes.

If you look at the percentage price/cost change between the two acquisitions, the difference is even more pronounced. Using the price indexes, you projected an increase from \$18,500 to \$21,275, or 15.0 percent. The offer increase was from \$18,500 to \$23,000, or about 24.3 percent. In this case, you might ask the offeror why the price/cost rose at a rate 62 percent higher than anticipated (24.3 is 62 percent larger than 15.0).

Do not attempt to determine whether a price or cost is reasonable based on this type of analysis alone. You must consider the entire contracting situation, including any differences in quantity, quality, delivery requirements, or other contract terms that might significantly affect price. However, the above analysis does raise concern about the reasonableness of the offer.

Note that the analysis above is based on 4-year old data. You should generally place less reliance on a comparison utilizing 4-year old data than you place on a comparison based on more current data.

1.4.2 Adjusting Price/Cost For Further Analysis

Inflation/Deflation May Obscure Trends. Often you will make a series of similar acquisitions over a period of time. Pricing trends may develop but they may be obscured by inflation/deflation. Adjusting prices for inflation/deflation will make it possible to more accurately identify and track these trends.

Steps in Using Price Indexes to Analyze Price/Cost Reasonableness. Adjustment for further analysis follows four steps similar to those used for data adjustment that are applied in preparation for direct comparison. The major difference is that several elements of cost/price data must be adjusted to a single time period. After adjustment, data is said to be in constant-year dollars.

Step 1. Collect available price/cost data.

Step 2. Select price indexes for adjusting price/cost data.

Step 3. Adjust prices/costs for inflation/deflation.

Step 4. Apply appropriate analysis technique(s).

Example of Using Price Indexes to Adjust Prices/ Costs for Further Analysis. To illustrate this analysis, consider an offer of \$22,500 each for five precision presses in 20X7.

Step 1. Collect Available Price/Cost Data. The organization has purchased five similar presses each year since 20X2. The historical unit prices are shown in Column D of the table below. While purchase quantity changes are not present in this situation, unit prices are used to limit the effect of quantity differences on trend analysis. In this case, the only apparent cost/price trend in the unadjusted data are the increasing prices.

Step 2. Select Price Indexes For Adjusting Price/Cost Data. Again, the Machinery and Equipment Index will be used. Annual indexes are presented in Column B of the table below.

Step 3. Adjust Prices/Costs For Inflation/Deflation. The adjustment calculation is presented in Column C of the table below. Each historical price is adjusted to an equivalent price in 20X7 dollars.

Adjustment For Further Analysis				
A	B	C	D	E
Year	Machinery and Equipment Index	Index Adjustment Calculation	Historical Prices	Adjusted Prices
20X2	100.0	121.9 100.0	\$17,391	\$21,200*

20X3	103.3	121.9 103.3	\$17,796	\$21,000
20X4	106.0	121.9 106.0	\$18,087	\$20,800
20X5	110.8	121.9 110.8	\$18,724	\$20,600
20X6	115.0	121.9 115.0	\$19,245	\$20,400
20X7	121.9	121.9 121.9	--	?

Step 4. Apply appropriate analysis technique(s). After the historical unit prices are adjusted to 20X7 dollars, a trend becomes obvious. In 20X7 dollars, prices have been dropping \$200 each year since 20X2. The obvious price estimate is \$20,200 for the 20X7 acquisition. That projection is based on the continuation of the historical trend. However, as with direct comparison, analysis based on historical price trends must consider any changes in the contracting situation and their possible affect on contract price. There may also be questions as to what has caused the trend and whether those forces will continue to cause price changes.

Most trends are not so obvious, even after prices have been adjusted to constant-year dollars. However, you can often apply techniques such as regression analysis or improvement curve analysis to identify clear estimating relationships.

1.5 Identifying Issues And Concerns

Questions to Consider in Analysis. As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever your analysis is based on data collected over time.

- Were prices/costs collected over time adjusted for inflation/deflation?

Inflation/deflation can mask underlying price changes. Price indexes should be used to more accurately identify and track any pricing trends.

- Is it reasonable to use the price index series selected?

The price index series selected for making the price/cost adjustment should be as closely related to the item being considered as possible. For example, you should not use the Consumer Price Index to adjust for changes in the price of complex industrial electronic equipment.

- Are adjustments calculated correctly?

Anyone can make a mistake in calculation. Assure that all adjustments are made correctly.

- Is the time period for the adjustment reasonable?

When adjusting historical prices for inflation, take care in selecting the period of adjustment. There are two basic methods that are used in adjusting costs/prices, period between acquisition dates and the period between delivery dates. The period between acquisition dates is most commonly used because purchase dates are typically more readily available. However, be careful if delivery schedules are substantially different.

- Is more than one adjustment made for the same inflation/ deflation?

For example, it is common for offerors to adjust supplier quotes to consider inflation/deflation between the time when the quote was obtained and the date that the product will be required. This is acceptable unless the supplier already considered the inflation/deflation in making the quote.

- How far into the future can you forecast?

You can forecast any period into the future as long as you have a reasonable index estimate. However, the price forecast risk increases as the risk of developing a reasonable index estimate increases. The farther into the future that you forecast, the greater the risk that the economic factors affecting the index will change.

2.0 Chapter Introduction

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In this chapter, you will learn to use cost-volume-profit analysis.

Assumptions . When you acquire supplies or services, you normally expect to pay a smaller price per unit as the purchase quantity increases. You expect contractors to have lower costs per unit as production quantity increases. This general expectation remains the same whether you are buying items specifically built for the Government or items that are mass-produced for a variety of commercial and Government customers. You can use cost-volume-profit analysis to analyze the natural relationship between cost, volume, and profit in pricing decisions. In cost-volume-profit analysis, you:

- Should consider only short-term operations. The short term may be defined as a period too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships.
- Assume that a straight line can reasonably be used in analysis. While actual price behavior may not follow a straight line, its use can closely approximate actual cost behavior in the short run.
 - If purchase volume moves outside the relevant range of the available data, the straight-line assumption and the accuracy of estimates become questionable.
 - If you know that product variable costs per unit are decreasing as quantity increases, consider using the log-linear improvement curve concept. Improvement curves are particularly useful in limited production situations where you can obtain cost/price information for all units sold.

Types of Cost . In the short run, costs can be of three general types:

- **Fixed Cost.** Total fixed costs remain constant as volume varies in the relevant range of production. Fixed cost per unit decreases as the cost is spread over an increasing number of units.

Examples include: Fire insurance, depreciation, facility rent, and property taxes.

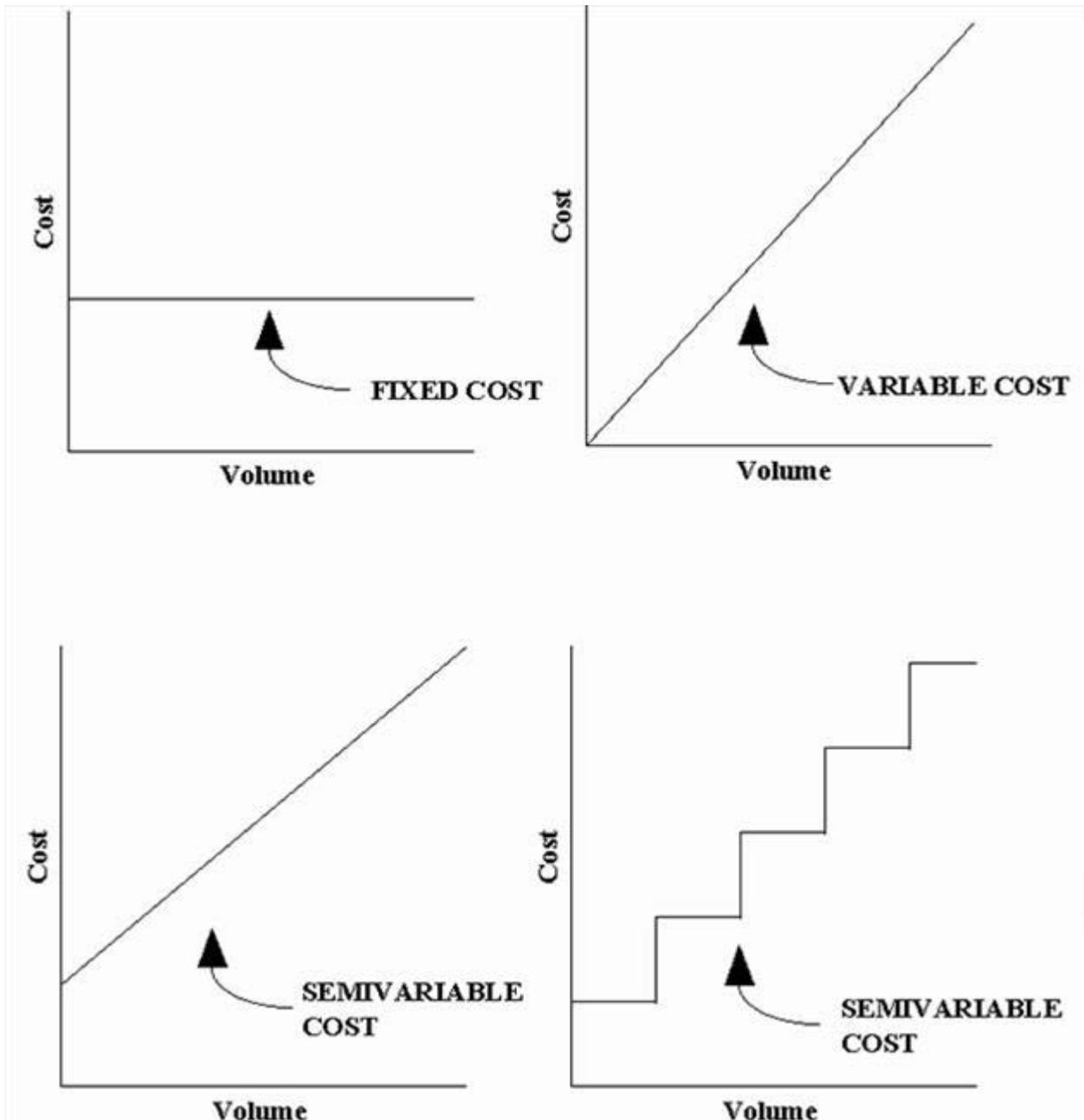
- **Variable Cost.** Variable cost per unit remains constant no matter how many units are made in the relevant range of production. Total variable cost increases as the number of units increases.

Examples include: Production material and labor. If no units are made, neither cost is necessary or incurred. However, each unit produced requires production material and labor.

- **Semi - variable Cost.** Semi - variable costs include both fixed and variable cost elements. Costs may increase in steps or increase relatively smoothly from a fixed base.

Examples include: Supervision and utilities, such as electricity, gas, and telephone. Supervision costs tend to increase in steps as a supervisor's span of control is reached. Utilities typically have a minimum service fee, with costs increasing relatively smoothly as more of the utility is used.

Graphic Depiction of Cost Behavior . The four graphs below illustrate the different types of cost behavior described above:



Profit . Profit is the difference between total cost and revenue. In cost-volume-profit analysis, a loss is expressed as a negative profit. Breaking even, which is neither profit nor loss, is a profit of zero dollars.

2.1 Identifying Situations For Use

Situations for Use . Cost-volume-profit analysis is an estimating concept that can be used in a variety of pricing situations. You can use the cost-volume relationship for:

- **Evaluating item price in price analysis.** Cost-volume-profit analysis assumes that total cost is composed of fixed and variable elements. This assumption can be used to explain price changes as well as cost changes. As the volume being acquired increases unit costs decline. As unit costs decline, the vendor can reduce prices and same make the same profit per unit.
- **Evaluating direct costs in pricing new contracts.** Quantity differences will often affect direct costs -- particularly direct material cost. Direct material requirements often include a fixed component for development or production operation set-up. As that direct cost is spread over an increasing volume unit costs should decline.
- **Evaluating direct costs in pricing contract changes.** How will an increase in contract effort increase contract price? Some costs will increase while others will not. The concepts of cost-volume-profit analysis can be an invaluable aid in considering the effect of the change on contract price.
- **Evaluating indirect costs.** The principles of cost-volume-profit analysis can be used in indirect cost analysis. Many indirect costs are fixed or semi - variable. As overall volume increases, indirect cost rates typically decline because fixed costs are spread over an increasing production volume.

2.2 Analyzing The Cost-Volume Relationship

This section examines algebraic and graphic analysis of the cost-volume relationship.

- 2.2.1 - [Algebraic Analysis](#)
- 2.2.2 - [Graphic Analysis](#)

2.2.1 Algebraic Analysis

Key Assumption . The assumption of linear cost behavior permits use of straight-line graphs and simple linear algebra in cost-volume analysis.

Calculating Total Cost Algebraically . Total cost is a semi - variable cost-some costs are fixed, some costs are variable, and others are semi - variable. In analysis, the fixed component of a semi - variable cost can be treated like any other fixed cost. The variable component can be treated like any other variable cost. As a result, we can say that:

Total Cost = Fixed Cost + Variable Cost

Using symbols:

$$C = F + V$$

Where:

C = Total cost

F = Fixed cost

V = Variable cost

Total variable cost depends on two elements:

Variable Cost = Variable Cost per Unit x Volume Produced

Using symbols:

$$V = V_U (Q)$$

Where:

V_U = Variable cost per unit

Q = Quantity (volume) produced

Substituting this variable cost information into the basic total cost equation, we have the equation used in cost-volume analysis:

$$C = F + V_U (Q)$$

Example of Calculating Total Cost Algebraically . If you know that fixed costs are \$500, variable cost per unit is \$10, and the volume produced is 1,000 units, you can calculate the total cost of production.

$$C = F + V_U (Q)$$

$$= \$500 + \$10 (1,000)$$

$$= \$500 + \$10,000$$

$$= \$10,500$$

Example of Calculating Variable Cost Algebraically . Given total cost and volume for two different levels of production, and using the straight-line assumption, you can calculate variable cost per unit.

Remember that:

- Fixed costs do NOT change no matter what the volume, as long as production remains within the relevant range of available cost information. Any change in total cost is the result of a change in total variable cost.
- Variable cost per unit does NOT change in the relevant range of production.

As a result, we can calculate variable cost per unit (V_U) using the following equation:

$$V_U = \text{Change in Total Cost} / \text{Change in Volume}$$

or

$$= (C_2 - C_1) / (Q_2 - Q_1)$$

Where:

C_1 = Total cost for Quantity 1

C_2 = Total cost for Quantity 2

Q_1 = Quantity 1

Q_2 = Quantity 2

You are analyzing an offeror's cost proposal. As part of the proposal the offeror shows that a supplier offered 5,000 units of a key part for \$60,000. The same quote offered 4,000 units for \$50,000. What is the apparent variable cost per unit?

$$\begin{aligned} V_U &= (C_2 - C_1) / (Q_2 - Q_1) \\ &= (\$60,000 - \$50,000) / (5,000 - 4,000) \end{aligned}$$

$$= \$10,000 - \$1,000$$

$$= \$10$$

Example of Calculating Fixed Cost Algebraically . If you know total cost and variable cost per unit for any quantity, you can calculate fixed cost using the basic total cost equation.

You are analyzing an offeror's cost proposal. As part of the proposal the offeror shows that a supplier offered 5,000 units of a key part for \$60,000. The apparent variable cost is \$10 per unit. What is the apparent fixed cost?

$$C = F + V_U (Q)$$

$$\$60,000 = F + \$10 (5,000)$$

$$\$60,000 - \$50,000 = F$$

$$\$10,000 = F$$

Developing an Estimating Equation . Now that you know that V_U is \$10 and F is \$10,000 you can substitute the values into the general total cost equation.

$$C = F + V_U (Q)$$

$$= \$10,000 + \$10 (Q)$$

You can use this equation to estimate the total cost of any volume in the relevant range between 4,000 and 5,000 units.

Using the Estimating Equation . Using the estimating equation for the relevant range, estimate the total cost of 4,400 units.

$$\begin{aligned} C &= \$10,000 + \$10 (Q) \\ &= \$10,000 + \$10 (4,400) \\ &= \$10,000 + \$44,000 \\ &= \$54,000 \end{aligned}$$

2.2.2 Graphic Analysis

Introduction to Graphic Analysis . When you only have two data points, you must generally assume a linear relationship. When you get more data, you can examine the data to determine if there is truly a linear relationship.

You should always graph the data before performing an algebraic analysis.

- Graphic analysis is the best way of developing an overall view of cost-volume relationship.
- Graphic analysis is useful in analyzing cost-volume relationships, particularly, when the cost and volume numbers involved are relatively small.
- Even when actual analysis is performed algebraically you can use graphs to demonstrate cost-volume analysis to others.

Steps of Graphic Analysis . There are four steps in using graph paper to analyze cost-volume relationships:

Step 1. Determine the scale that you will use. Volume is considered the independent variable and will be graphed on the horizontal axis. Cost is considered the dependent variable and will be graphed on the vertical axis. The scales on the two axes do not have to be the same. However, on each axis one block must represent the same amount of change as every other block of the same size on that axis. Each scale should be large enough to permit analysis, and small enough to permit the graphing of all available data and anticipated data estimates.

Step 2. Plot the available cost-volume data. Find the volume given for one of the data points on the horizontal axis. Draw an imaginary vertical line from that point. Find the related cost on the vertical axis and draw an imaginary horizontal line from that point. The point where the two lines intersect represents the cost for the given volume. (If you do not feel comfortable with imaginary lines you may draw dotted lines to locate the intersection.) Repeat this step for each data point.

Step 3. Fit a straight line to the data. In this section of text, all data points will fall on a straight line. All that you have to do to fit a straight line is connect the data points. Most analysts use regression analysis to fit a straight line when all points do not fall on the line.

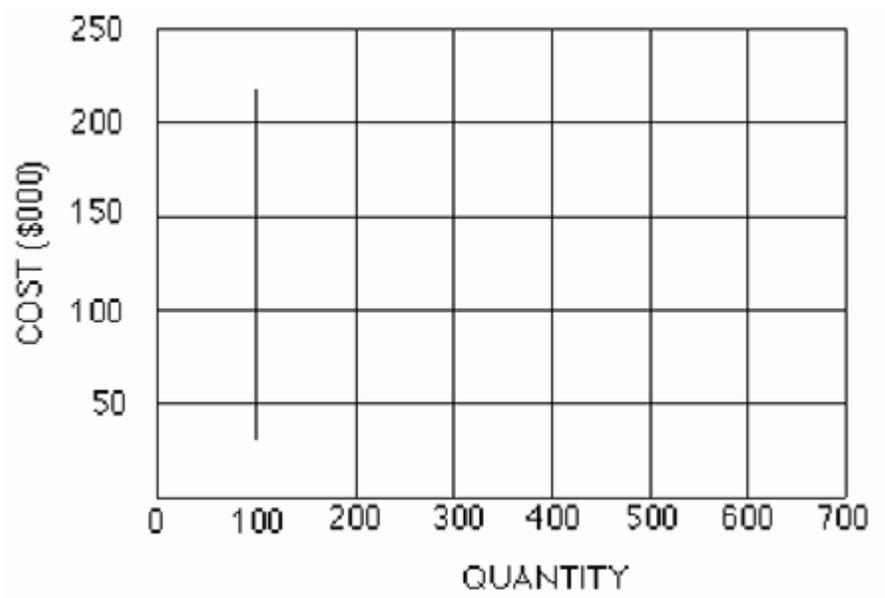
Step 4. Estimate the cost for a given volume. Draw an imaginary vertical line from the given volume to the point where it intersects the straight line that you fit to the data points. Then move horizontally until you intersect the vertical axis. That point is the graphic estimate of the cost for the given volume of the item.

Example of Graphic Analysis . The four steps of cost-volume-profit analysis can be used to graph and analyze any cost-volume relationship. Assume that you have been asked to estimate the cost of 400 units given the following data:

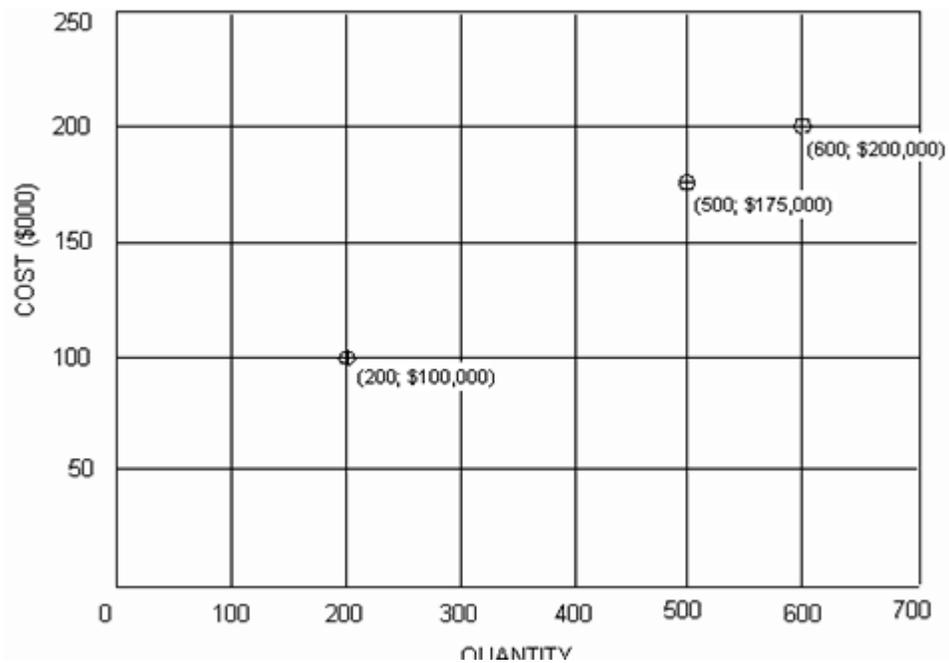
Units	Cost
200	\$100,000

500	\$175,000
600	\$200,000

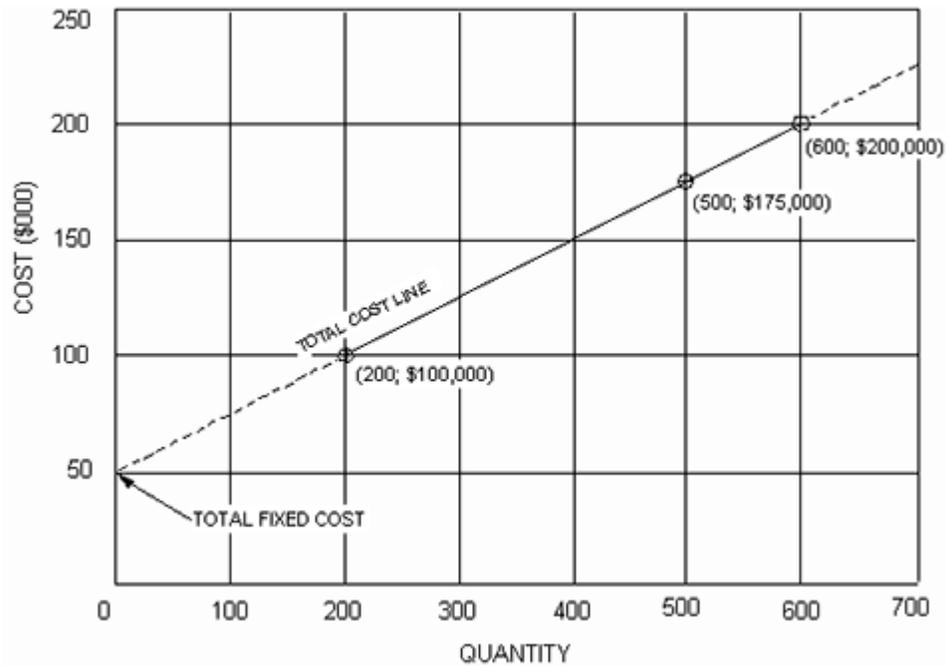
Step 1. Determine the scale that you will use .



Step 2. Plot the available cost-volume data.

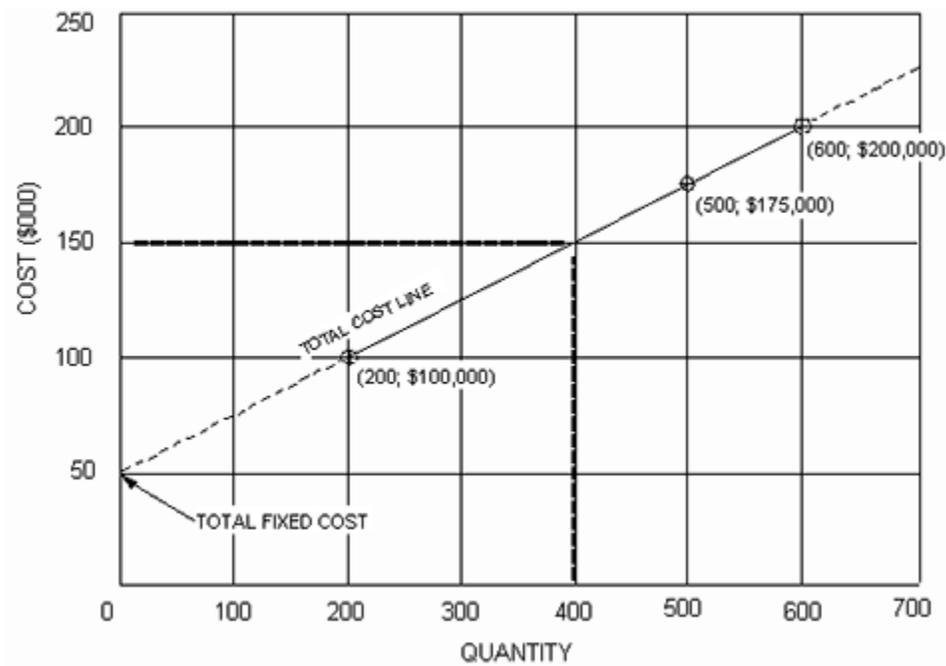


Step 3. Fit a straight line to the data.



Step 4. Estimate the cost for a given volume. From the graph, you can estimate that the total cost of 400 units will be \$150,000.

In addition you can also estimate fixed cost. The cost of making zero units, \$50,000, is the fixed cost for this set of data.



You can use any two points on the line to calculate the equation of the line. Follow the procedures in the section on Algebraic Analysis.

2.3 Analyzing The Price-Volume Relationship

Analysis Situations . In situations where you do not have offeror cost data, you can use the principles of the cost-volume relationship in price-volume analysis. Price-volume analysis is based on pricing information that is typically available to the Government negotiator.

Quantity Price Discounts . Unit prices normally decline as volume increases, primarily because fixed costs are being divided by an increasing number of units. Buyers see these price reductions with increasing volume in the form of quantity discounts.

Quantity discounts complicate the pricing decision, because a price that is reasonable for one volume may not be reasonable for a different volume.

Offers quote quantity discounts because their costs per unit declines as volume increases. As a result, even though offered prices include profit, you can use the cost-volume equation to estimate prices at different quantities.

Steps for Estimating a Quantity Price Discount . Estimating a quantity price discount is a 4-step process:

Step 1. Calculate the variable element.

Step 2. Calculate the fixed element using data from the available data points.

Step 3. Develop an estimating equation.

Step 4. Estimate the price for a given quantity

Example of Estimating a Quantity Price Discount . You know that the unit price for 100 units is \$3,000 and the unit price for 500 units is \$2,500. You are about to purchase 250 units. How can you use the available information to estimate the price for 250 units?

Approach this question just as though you were being asked to estimate cost given information for two different quantities. The only difference is that you use price data instead of cost data in the analysis.

Step 1. Calculate the variable element.

$$V_U = (C_2 - C_1) / (Q_2 - Q_1)$$
$$= (\$1,250,000 - \$300,000) / (500 - 100)$$

$$= \$950,000 / 400$$

$$= \$2,375$$

Step 2. Calculate the fixed element using data from the available data points.

$$C = F + V_U (Q)$$

$$\$300,000 = F + \$2,375 (100)$$

$$\$300,000 - \$2,375 (100) = F$$

$$\$300,000 - \$237,500 = F$$

$$\$62,500 = F$$

Step 3. Develop an estimating equation.

$$C = F + V_U (Q)$$

$$= \$62,500 + \$2,375(Q)$$

Step 4. Estimate the price for 250 units.

$$C = \$62,500 + \$2,375 (Q)$$

$$= \$62,500 + \$2,375 (250)$$

$$= \$62,500 + \$593,750$$

$$= \$656,250 \text{ total price and a } \$2,625 \text{ unit price}$$

2.4 Analyzing The Cost-Volume-Profit Relationship

Until now, we have only looked at the cost-volume or price-volume relationship. Now, we are going to expand that relationship to consider the relationship between cost, volume, and profit .

Cost-Volume-Profit Equation . The revenue taken in by a firm is equal to cost plus profit. That can be written:

$$\text{Revenue} = \text{Total Cost} + \text{Profit}$$

We have already seen that total cost (C) is:

$$C = F + V_U(Q)$$

Using this information, we can rewrite the revenue equation as:

$$\text{Revenue} = F + V_U(Q) + \text{Profit}$$

In the cost-volume-profit equation, **profit can be positive, negative, or zero** . If profit is negative, we normally refer to it as a loss. If profit is zero, the firm is breaking even with no profit or loss. If we let P stand for profit, we can write the equation:

$$\text{Revenue} = F + V_U(Q) + P$$

Revenue is equal to selling price per unit (R_U) multiplied by volume.

$$\text{Revenue} = R_U(Q)$$

If we assume that the firm makes all the units that it sells, and sells all the units that it makes, we can complete the cost-volume-profit equation:

$$R_U(Q) = F + V_U(Q) + P$$

Application of the Cost-Volume-Profit Equation . This equation and limited knowledge of a contractor's cost structure can provide you with extremely valuable information on the effect purchase decisions can have on a firm's profitability.

Using the Cost-Volume-Profit Equation to Estimate Selling Price . Given the following product information, a firm prepared an offer for an indefinite quantity contract with the Government for a new product developed by the firm. There are no other customers for the product. In developing the offer unit price estimate (R_U), the firm used its estimated costs and its best estimate of the quantity that it would sell under the contract.

$$\text{Fixed Cost} = \$10,000$$

$$\text{Variable Cost per Unit} = \$20$$

$$\text{Contract Minimum Quantity} = 3,000 \text{ units}$$

$$\text{Contract Maximum Quantity} = 6,000 \text{ units}$$

$$\text{Firm's Best Estimate of Quantity} = 5,000 \text{ units}$$

Target Profit = \$5,000

$$R_U(Q) = F + V_U(Q) + P$$

$$R_U(5,000) = \$10,000 + \$20(5,000) + \$5,000$$

$$R_U(5,000) = \$10,000 + \$100,000 + \$5,000$$

$$R_U(5,000) = \$115,000$$

$$R_U = \$115,000 / 5,000$$

$$R_U = \$23.00$$

Using the Cost-Volume-Profit Equation to Estimate Profit . Managers of the firm wanted to know how profits would be affected if it actually sold the maximum quantity (6000 units) at \$23.00 per unit.

$$R_U(Q) = F + V_U(Q) + P$$

$$\$23(6,000) = \$10,000 + \$20(6,000) + P$$

$$\$138,000 = \$10,000 + \$120,000 + P$$

$$\$138,000 = \$130,000 + P$$

$$\$138,000 - \$130,000 = P$$

$$\$8,000 = P$$

If the firm sells 6,000 units at \$23.00 per unit, profit will be \$8,000. That is a \$3,000 increase from the original \$5,000 target profit, or an increase of 60 percent. Note that the firm's profit would increase solely because sales were higher than estimated.

Managers were even more concerned about how profits would be affected if they only sold the minimum quantity (4000 units) at \$23.00 per unit.

$$R_U(Q) = F + V_U(Q) + P$$

$$\$23(4,000) = \$10,000 + \$20(4,000) + P$$

$$\$92,000 = \$10,000 + \$80,000 + P$$

$$\$92,000 = \$90,000 + P$$

$$\$2,000 = P$$

If the firm sells 4,000 at \$23.00 per unit, profit will be \$2,000. That is \$3,000 less than the original \$5,000 target profit. Note that the firm's profit would decrease solely because sales were lower than estimated.

Using the Cost-Volume-Profit Equation to Estimate Break-Even Sales . In a final effort to analyze the risk to the firm under the proposed indefinite deliver contract, managers wanted to know the level of sales that would be required for the firm to break even (zero profit).

$$R_U(Q) = F + V_U(Q) + P$$

$$\$23(Q) = \$10,000 + \$20(Q) + 0$$

$$(\$23 - \$20)(Q) = \$10,000$$

$$\$3(Q) = \$10,000$$

$$Q = \$10,000 / \$3$$

$$Q = 3,333.33 \text{ units}$$

The calculations show that the firm would break even at 3,333.33 units. Assuming that the firm could not sell .33 units, the firm must sell 3,334 units to assure that all costs are covered. Selling 3,333 units would still result in a \$.10 loss.

Contribution Income . The difference between revenue and variable cost is contribution income (I). The term contribution income comes from the contribution made to covering fixed costs and profit. If contribution income is positive, increasing sales will increase profits or reduce losses. If contribution income is negative, increasing sales will reduce profits or create greater losses.

$$\text{Contribution Income} = \text{Revenue} - \text{Variable Cost}$$

Using symbols:

$$I = R_U(Q) - V_U(Q) \text{ or}$$

$$I = (R_U - V_U)(Q)$$

Knowledge of a contractor's cost structure and contribution income can be valuable in analysis of proposed costs.

Contribution Income Example . In evaluating an offeror's proposal for 500 units at \$900 each, your analysis reveals the following cost structure:

Fixed Cost = \$100,000
Variable Cost per Unit = \$1,000

How would this affect your analysis of contract risk?

$$\begin{aligned} I &= (R_U - V_U) (Q) \\ &= (\$900 - \$1,000) (500) \\ &= -\$100 (500) \\ &= -\$50,000 \end{aligned}$$

The contribution income from the sale is a negative \$50,000. The firm would be substantially worse off for having made the sale. Unless the firm can offer a positive rationale for such a pricing decision, you must consider pricing as an important factor as you analyze the risk of contract performance.

2.5 Identifying Issues And Concerns

Questions to Consider in Analysis . As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use cost-volume-profit analysis concepts.

- ***Has the contractor's cost structure changed substantially?***

Application of cost-volume-profit analysis assumes that the period covered by the analysis is too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships. If the contractor has substantially changed its cost structure, your ability to use cost-volume-profit analysis may be limited. Examples of possible changes include:

- - Downsizing to reduce fixed costs; and
 - Increased investment in automated equipment to reduce variable costs of labor and material.
- ***Is the straight-line assumption reasonable?***

The cost-volume-profit relationship is not usually a straight- line relationship. Instead, it is a curvilinear relationship. A straight-line analysis works as long as the straight line is a good approximation of the cost-volume-profit relationship. Most computer programs designed to fit a

straight-line to a set of data provide measures of how well the line fits the data. For example, a regression program will usually provide the coefficient of determination (r^2).

- *Are current volume estimates within the relevant range of available data?*

If the current business volume is substantially higher or lower than the volumes used to develop the cost-volume-profit equation, the results may be quite unreliable. The contractor should be expected to change the way it does business and its cost structure if volume increases or decreases substantially.

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- 3.1 - [Identifying Situations For Use](#)
- 3.2 - [Measuring Central Tendency](#)
- 3.3 - [Measuring Dispersion](#)
- 3.4 - [Establishing A Confidence Interval](#)
- 3.5 - [Using Stratified Sampling](#)
- 3.6 - [Identifying Issues and Concerns](#)

3.0 - Chapter Introduction

In this chapter, you will learn to use descriptive statistics to organize, summarize, analyze, and interpret data for contract pricing.

Categories of Statistics . Statistics is a science which involves collecting, organizing, summarizing, analyzing, and interpreting data in order to facilitate the decision-making process. These data can be facts, measurements, or observations. For example, the inflation rate for various commodity groups is a statistic which is very important in contract pricing. Statistics can be classified into two broad categories:

- **Descriptive Statistics.** Descriptive statistics include a large variety of methods for summarizing or describing a set of numbers. These methods may involve computational or graphical analysis. For example, price index numbers are one example of a descriptive statistic. The measures of central tendency and dispersion presented in this chapter are also descriptive statistics, because they describe the nature of the data collected.
- **Inferential Statistics.** Inferential or inductive statistics are methods of using a sample data taken from a statistical population to make actual decisions, predictions, and generalizations related to a problem of interest. For example, in contract pricing, we can use stratified sampling of a proposed bill of materials to infer the degree it is overpriced or under-priced.

Populations and Samples . The terms population and sample are used throughout any discussion of statistics.

- **Population.** A population is the set of all possible observations of a phenomenon with which we are concerned. For example, all the line items in a bill of materials would constitute a population. A numerical characteristic of a population is called a parameter.
- **Sample.** A sample is a subset of the population of interest that is selected in order to make some inference about the whole population. For example, a group of line items randomly selected from a bill of materials for analysis would constitute a sample. A numerical characteristic of a sample is called a statistic.

In contract pricing, you will most likely use statistics because you do not have complete knowledge of the population or you do not have the resources needed to examine the population data. Because most pricing applications involve the use of sample data, this chapter will concentrate on statistical analysis using sample data. If you are interested in learning about descriptive or inferential analysis of numerical population data, consult a college level statistics text.

Measure of Reliability . Since a sample contains only a portion of observations in the population, there is always a chance that our inference or generalization about the population will be inaccurate. Therefore, our inference should be accompanied by a measure of reliability. For example, let's assume that we are 90 percent sure that the average item in a bill of materials should cost 85 percent of what the contractor has proposed plus or minus 3 percent. The 3 percent is simply a boundary for our prediction error and it means there is a 90 percent probability that the error of our prediction is not likely to exceed 3 percent.

3.1 - Identifying Situations For Use

Situations for Use . Statistical analysis can be invaluable to you in:

- **Developing Government objectives for contract prices based on historical values.** Historical costs or prices are often used as a basis for prospective contract pricing. When several historical data points are available, you can use statistical analysis to evaluate the historical data in making estimates for the future. For example, you might estimate the production equipment set-up time based on average historical set-up times.
- **Developing minimum and maximum price positions for negotiations.** As you prepare your negotiation objective, you can also use statistical analysis to develop minimum and maximum positions through analysis of risk. For example, if you develop an objective of future production set-up time based on the average of historical experience, that average is a point estimate calculated from many observations. If all the historical observations are close to the point estimate, you should feel confident that actual set-up time will be close to the estimate. As the differences between the individual historical observations and the point estimate increase, the risk that the future value will be substantially different than the point estimate also increases. You can use statistical analysis to assess the cost risk involved and use that assessment in developing your minimum and maximum negotiation positions.

- **Developing an estimate of risk for consideration in contract type selection.** As described above, you can use statistical analysis to analyze contractor cost risk. In addition to using that analysis in developing your minimum and maximum negotiation positions you can use it in contract type selection. For example, if the risk is so large that a firm fixed-price providing reasonable protection to the contractor could also result in a wind-fall profit, you should consider an incentive or cost-reimbursement contract instead.
 - **Developing an estimate of risk for consideration in profit/fee analysis.** An analysis of cost risk is also an important element in establishing contract profit/fee objectives. The greater the dispersion of historical cost data, the greater the risk in prospective contract pricing. As contractor cost risk increases, contract profit/fee should also increase.
 - **Streamlining the evaluation of a large quantity of data without sacrificing quality.** Statistical sampling is particularly useful in the analysis of a large bill of materials. The stratified sampling techniques presented in this chapter allow you to:
 - Examine 100 percent of the items with the greatest potential for cost reduction; and
 - Use random sampling to assure that there is no general pattern of overpricing smaller value items.
 - The underlying assumption of random sampling is that a sample is representative of the population from which it is drawn.
 - If the sample is fairly priced, the entire stratum is assumed to be fairly priced; if the sample is overpriced, the entire stratum is assumed to be proportionately overpriced.
-

3.2 - Measuring Central Tendency

Measures of Central Tendency . You are about to prepare a solicitation for a product that your office has acquired several times before. Before you begin, you want to know what your office has historically paid for the product. You could rely exclusively on the last price paid, or you could collect data from the last several acquisitions. An array of data from several acquisitions will likely mean little without some statistical analysis.

To get a clearer picture of this array of data, you would likely want to calculate some measure of central tendency. A measure of central tendency is the central value around which data observations (e.g., historical prices) tend to cluster. It is the central value of the distribution. This section will examine calculation of the three most common and useful measures of central tendency:

- Arithmetic mean;
- Median; and
- Mode.

Calculating the Arithmetic Mean . The arithmetic mean (or simply the mean or average) is the measure of central tendency most commonly used in contract pricing. To calculate the mean,

sum all observations in a set of data and divide by the total number of observations involved. The formula for this calculation is:

$$\bar{X} = \frac{\sum X}{n}$$

Where:

\bar{X} = Sample mean

S = Summation of all the variables that follow the symbol (e.g., $\sum X$ represents the sum of all X values)

X = Value for an observation of the variable X

n = Total number of observations in the sample

For example: Suppose you are trying to determine the production lead time (PLT) for an electronic component using the following sample data:

Item	1	2	3	4	5	6	7	8
PLT (Months)	9	7	9	9	11	8	11	8

$$\begin{aligned} \bar{X} &= \frac{\sum X}{n} \\ &= \frac{9 + 7 + 9 + 9 + 11 + 8 + 11 + 8}{8} \\ &= \frac{72}{8} \\ &= 9 \text{ months} \end{aligned}$$

Calculating the Median . The median is the middle value of a data set when the observations are arrayed from the lowest to the highest (or from the highest to the lowest). If the data set contains an even number of observations, the median is the arithmetic mean of the two middle observations.

It is often used to measure central tendency when a few observations might pull the measure from the center of the remaining data. For example, average housing value in an area is commonly calculated using the median, because a few extremely high-priced homes could result in a mean that presents an overvalued picture of the average home price.

For the PLT example: You could array the data from lowest to highest:

X	7	8	8	9	9	9	11	11
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Since there is an even number of observations, calculate median using the arithmetic mean of the two middle observations:

$$\begin{aligned} Md &= \frac{9 + 9}{2} \\ &= 9 \text{ months} \end{aligned}$$

Calculating the Mode . The mode is the observed value that occurs most often in the data set (i.e. the value with the highest frequency).

It is often used to estimate which specific value is most likely to occur in the future. However, a data set may have more than one mode. A data set is described as:

- Unimodal if it has only one mode.
- Bimodal if it has two modes.
- Multimodal if it has more than two modes.

For the PLT example: The mode is nine, because nine occurs three times, once more than any other value.

3.3 - Measuring Dispersion

Measures of Dispersion . As described in the previous section, the mean is the measure of central tendency most commonly used in contract pricing. Though the mean for a data set is a value around which the other values tend to cluster, it conveys no indication of the closeness of the clustering (that is, the dispersion). All observations could be close to the mean or far away.

If you want an indication of how closely these other values are clustered around the mean, you must look beyond measures of central tendency to measures of dispersion. This section will examine:

- Several measures of absolute dispersion commonly used to describe the variation within a data set -- the range, mean absolute deviation, variance, and standard deviation.
- One measure of relative dispersion-- the coefficient of variation.

Assume that you have the following scrap rate data for two contractor departments:

	Contractor Scrap Rate Data	
Month	Dept. A, Fabrication	Dept. B, Assembly

February	.065	.050
March	.035	.048
April	.042	.052
May	.058	.053
June	.032	.048
July	.068	.049
Total	.300	.300
Mean	.050	.050

The mean scrap rate for both departments is the same -- 5 percent. However, the monthly scrap rates in Department B show less variation (dispersion) around the mean. As a result, you would probably feel more comfortable forecasting a scrap rate of 5 percent for Department B than you would for Department A.

Differences in dispersion will not always be so obvious. The remainder of this section will demonstrate how you can quantify dispersion using the five measures identified above.

Calculating the Range . Probably the quickest and easiest measure of dispersion to calculate is the range. The range of a set of data is the difference between the highest and lowest observed values. The higher the range, the greater the amount of variation in a data set.

$$R = H - L$$

Where:

R = Range

H = Highest observed value in the data set

L = Lowest observed value in the data set

Calculating the Range for the Scrap-Rate Example . By comparing the range for Department A scrap-rate data with the range for Department B, you can easily determine that the historical data from Department A shows greater dispersion.

$$\begin{aligned} R_{\text{DEPT A}} &= .068 - .032 \\ &= .036 \end{aligned}$$

$$\begin{aligned} R_{\text{DEPT B}} &= .053 - .048 \\ &= .005 \end{aligned}$$

Mean Absolute Deviation . The mean absolute deviation (MAD) is the average absolute difference between the observed values and the arithmetic mean (average) for all values in the data set.

If you subtracted the mean from each observation, some answers would be positive and some negative. The sum of all the deviations (differences) will always be zero. That tells you nothing about how far the average observation is from the mean. To make that computation, you can use the absolute difference between each observation and the mean. An absolute difference is the difference without consideration of sign and all absolute values are written as positive numbers. If the average is eight and the observed value is six, the calculated difference is a negative two ($6 - 8 = -2$), but the absolute difference would be two (without the negative sign).

Note : Absolute values are identified using a vertical line before and after the value (e.g., $|X|$ identifies the absolute value of X).

To compute the MAD, use the following 5-step process:

Step 1 . Calculate the arithmetic mean of the data set.

Step 2 . Calculate the deviation (difference) between each observation and the mean of the data set.

Step 3 . Convert each deviation to its absolute value (i.e., its value without considering the sign of the deviation).

Step 4 . Sum the absolute deviations.

Step 5 . Divide the total absolute deviation by the number of observations in the data set.

$$\text{MAD} = \frac{\sum |X - \bar{X}|}{n}$$

Calculating the Mean Absolute Deviation for the Scrap-Rate Example . We can use the 5-step process described above to calculate the scrap-rate MAD values for Departments A and B of the scrap-rate example.

Calculate the MAD for Department A:

Step 1. Calculate the arithmetic mean of the data set. We have already calculated the mean rate for Department A of the scrap-rate example -- .05.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set ($X - \bar{X}$).

Department A (Fabrication)		
X	\bar{X}	$X - \bar{X}$
.065	.050	.015

.035	.050	-.015
.042	.050	-.008
.058	.050	.008
.032	.050	-.018
.068	.050	.018

Step 3. Convert each deviation to its absolute value $(|x - \bar{x}|)$.

Department A (Fabrication)			
X	\bar{x}	$x - \bar{x}$	$ x - \bar{x} $
.065	.050	.015	.015
.035	.050	-.015	.015
.042	.050	-.008	.008
.058	.050	.008	.008
.032	.050	-.018	.018
.068	.050	.018	.018

Step 4. Sum the absolute deviations $(\sum |x - \bar{x}|)$.

Department A Fabrication)			
X	\bar{x}	$x - \bar{x}$	$ x - \bar{x} $
.065	.050	.015	.015
.035	.050	-.015	.015
.042	.050	-.008	.008
.058	.050	.008	.008
.032	.050	-.018	.018
.068	.050	.018	.018
Total			.082

Step 5. Divide the total absolute deviation by the number of observations in the data set.

$$MAD = \frac{\sum |X - \bar{X}|}{n}$$

$$\begin{aligned} MAD_{\text{Dept A}} &= \frac{.082}{6} \\ &= .014 \end{aligned}$$

Calculate the MAD for Department B:

Step 1. Calculate the arithmetic mean of the data set. We have also calculated the mean rate for Department B of the scrap-rate example -- .05.

Steps 2 - 4. Calculate the deviation between each observation and the mean of the data set; convert the deviation to its absolute value; and sum the absolute deviations. The following table demonstrates the three steps required to calculate the total absolute deviation for Department B:

Department B (Assembly)			
X	\bar{X}	$x - \bar{X}$	$ x - \bar{X} $
.050	.050	.000	.000
.048	.050	-.002	.002
.052	.050	.002	.002
.053	.050	.003	.003
.048	.050	-.002	.002
.049	.050	-.001	.001
Total			.010

Step 5. Divide the total absolute deviation by the number of observations in the data set.

$$MAD = \frac{\sum |X - \bar{X}|}{n}$$

$$\begin{aligned} MAD_{\text{Dept B}} &= \frac{.010}{6} \\ &= .002 \end{aligned}$$

Compare MAD values for Department A and Department B:

The MAD for Department A is .014; the MAD for Department B is .002. Note that the MAD for Department B is much smaller than the MAD for Department A. This comparison once again confirms that there is less dispersion in the observations from Department B.

Calculating the Variance . Variance is one of the two most popular measures of dispersion (the other is the standard deviation which is described below). The variance of a sample is the average of the squared deviations between each observation and the mean.

However, statisticians have determined when you have a relatively small sample, you can get a better estimate of the true population variance if you calculate variance by dividing the sum of the squared deviations by $n - 1$, instead of n .

$$S^2 = \frac{\sum (X - \bar{X})^2}{n - 1}$$

The term, $n - 1$, is known as the number of degrees of freedom that can be used to estimate population variance.

This adjustment is necessary, because samples are usually more alike than the populations from which they are taken. Without this adjustment, the sample variance is likely to underestimate the true variation in the population. Division by $n - 1$ in a sense artificially inflates the sample variance but in so doing, it makes the sample variance a better estimator of the population variance. As the sample size increases, the relative affect of this adjustment decreases (e.g., dividing by four rather than five will have a greater affect on the quotient than dividing by 29 instead of 30).

To compute the variance, use this 5-step process:

Step 1. Calculate the arithmetic mean of the data set.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set.

Step 3. Square each deviation.

Step 4. Sum the squared deviations.

Step 5. Divide the sum of the squared deviations by $n-1$.

Calculate the variance for Department A:

Step 1. Calculate the arithmetic mean of the data set. We have already calculated the mean rate for Department A of the scrap-rate example -- .05.

Step 2. Calculate the deviation (difference) between each observation and the mean of the data set ($x - \bar{x}$). The deviations for Department A are the same as we calculated in calculating the mean absolute deviation.

Department A (Fabrication)		
x	\bar{x}	$x - \bar{x}$
.065	.050	.015
.035	.050	-.015
.042	.050	-.008
.058	.050	.008
.032	.050	-.018
.068	.050	.018

Step 3. Square each deviation ($x - \bar{x}$)².

Department A (Fabrication)			
x	\bar{x}	$x - \bar{x}$	$ x - \bar{x} $
.065	.050	.015	.000225
.035	.050	-.015	.000225
.042	.050	-.008	.000064
.058	.050	.008	.000064
.032	.050	-.018	.000324
.068	.050	.018	.000324

Step 4. Sum the total absolute deviations $[\sum(x - \bar{x})^2]$.

Department A (Fabrication)			
x	\bar{x}	$x - \bar{x}$	$ x - \bar{x} $
.065	.050	.015	.000225
.035	.050	-.015	.000225
.042	.050	-.008	.000064
.058	.050	.008	.000064

.032	.050	-.018	.000324
.068	.050	.018	.000324
Total			.001226

Step 5. Divide the sum of the squared deviations by n-1.

$$S^2 = \frac{\sum (X - \bar{X})^2}{n - 1}$$

$$\begin{aligned} S_{\text{Dept A}}^2 &= \frac{.001226}{6 - 1} \\ &= \frac{.001226}{5} \\ &= .000245 \end{aligned}$$

Calculate the variance for Department B

Step 1. Calculate the arithmetic mean of the data set. We have also calculated the mean rate for Department B of the scrap-rate example -- .05.

Steps 2 - 4. Calculate the deviation between each observation and the mean of the data set; convert the deviation to its absolute value; and sum the absolute deviations. The following table demonstrates the three steps required to calculate the total absolute deviation for Department B:

Department B (Assembly)			
X	\bar{X}	$x - \bar{X}$	$(x - \bar{X})^2$
.050	.050	.000	.000000
.048	.050	-.002	.000004
.052	.050	.002	.000004
.053	.050	.003	.000009
.048	.050	-.002	.000004
.049	.050	-.001	.000001

Step 5. Divide the sum of the squared deviations by n-1.

$$S^2 = \frac{\sum (X - \bar{X})^2}{n - 1}$$

$$S_{\text{Dept A}}^2 = \frac{.000022}{6 - 1}$$

$$= \frac{.000022}{5}$$

$$= .000004$$

Compare variances for Department A and Department B:

The variance for Department A is .000245; the variance for Department B is .000004. Once again, the variance comparison confirms that there is less dispersion in the observations from Department B.

Concerns About Using the Variance as a Measure of Dispersion . There are two concerns commonly raised about using the variance as a measure of dispersion:

- As the deviations between the observations and the mean grow, the variation grows much faster, because all the deviations are squared in variance calculation.
- The variance is in a different denomination than the values of the data set. For example, if the basic values are measured in feet, the variance is measured in square feet; if the basic values are measured in terms of dollars, the variance is measured in terms of "square dollars."

Calculating the Standard Deviation . You can eliminate these two common concerns by using the standard deviation -- the square root of the variance.

$$S = \sqrt{S^2}$$

For example: You can calculate the standard deviation for Departments A and B of the scrap-rate example:

$$S_{\text{Dept A}} = \sqrt{.000245}$$

$$= .015652$$

$$S_{\text{Dept B}} = \sqrt{.000004}$$

$$= .002000$$

The standard deviation for Departments A and B of the scrap-rate example yields a standard deviation of .015652 for Department A, and .002000 for Department B.

Note: Both the variance and the standard deviation give increasing weight to observations that are further away from the mean. Because all values are squared, a single observation that is far from the mean can substantially affect both the variance and the standard deviation.

Empirical Rule . The standard deviation has one characteristic that makes it extremely valuable in statistical analysis. In a distribution of observations that is approximately symmetrical (normal):

- The interval 1S includes approximately 68 percent of the total observations in the population.
- The interval 2S includes approximately 95 percent of the total observations in the population.
- The interval 3S includes approximately 99.7 percent of the total observations in the population.

This relationship is actually a finding based on analysis of the normal distribution (bell shaped curve) that will be presented later in the chapter.

Coefficient of Variation . Thus far we have only compared two samples with equal means. In that situation, the smaller the standard deviation, the smaller the relative dispersion in the sample observations. However, that is not necessarily true when the means of two samples are not equal.

If the means are not equal, you need a measure of relative dispersion. The coefficient of variation (CV) is such a measure.

$$CV = \frac{S}{\bar{X}}$$

For example: Which of the following samples has more relative variation?

Sample C: $\bar{X} = 25$ $S = 5$

Sample D: $\bar{X} = 100$ $S = 10$

Calculate CV for Sample C:

$$\begin{aligned} CV_C &= \frac{5}{25} \\ &= .20 \end{aligned}$$

Calculate CV for Sample D:

$$\begin{aligned} CV_D &= \frac{10}{100} \\ &= .10 \end{aligned}$$

Compare the two CV values:

Even though the standard deviation for Sample D is twice as large as the standard deviation for Sample C, the CV values demonstrate that Sample D exhibits less relative variation. This is true because the mean for Sample D is so much larger than the mean for Sample C.

Note: We could calculate CV for the scrap-rate example, but such a calculation is not necessary because the means of the two samples are equal.

3.4 Establishing a Confidence Interval

Confidence Interval . Each time you take a sample from a population of values you can calculate a mean and a standard deviation. Even if all the samples are the same size and taken using the same random procedures, it is unlikely that every sample will have the same mean and standard deviation. However, if you could collect all possible samples from the normally distributed population and calculate the mean value for all the sample means, the result would be equal to the population mean. In statistical terminology, the mean of the sampling distribution is equal to the population mean.

You can combine the sample mean and sample standard deviation with an understanding of the shape of distribution of sample means to develop a **confidence interval -- a probability statement about an interval which is likely to contain the true population mean.**

For example: Suppose that you are preparing a solicitation for an indefinite-quantity transmission overhaul contract to support a fleet of 300 light utility trucks. You believe that you can develop an accurate estimate of the number of transmissions that will require a major overhaul during the contract period, if you can determine the date of the last major overhaul for each vehicle transmission and estimate the period between overhauls. You select a simple random sample (without replacement) of 25 vehicle maintenance records from the 300 fleet vehicle maintenance records. Analyzing the sample, you find that the mean time between overhauls is 38 months and the sample standard deviation (S) is 4 months. Based on this analysis, your point estimate of the average transmission life for all vehicles in the fleet (the population mean) is 38 months. But you want to establish reasonable estimates of the minimum and maximum number of repairs that will be required during the contract period. You want to be able to state that you are 90% confident that the average fleet transmission life is within a defined range (e.g., between 36 and 40 months).

To make this type of statement, you need to establish a confidence interval. You can establish a confidence interval using the sample mean, the standard error of the mean, and an understanding of the normal probability distribution and the t distribution.

Standard Error of the Mean . If the population is normally distributed, the standard error of the mean is equal to the population standard deviation divided by the square root of sample size. Since we normally do not know the population standard deviation, we normally use the sample standard deviation to estimate the population standard deviation.

$$S_{\bar{x}} = \frac{S}{\sqrt{n}}$$

Though the population mean and the population standard deviation are not normally known, we assume that cost or pricing information is normally distributed. This is a critical assumption because it allows us to construct confidence intervals (negotiation ranges) around point estimates (Government objectives).

Calculate the Standard Error of the Mean for the Transmission Example.

Remember that: $S = 4$ months

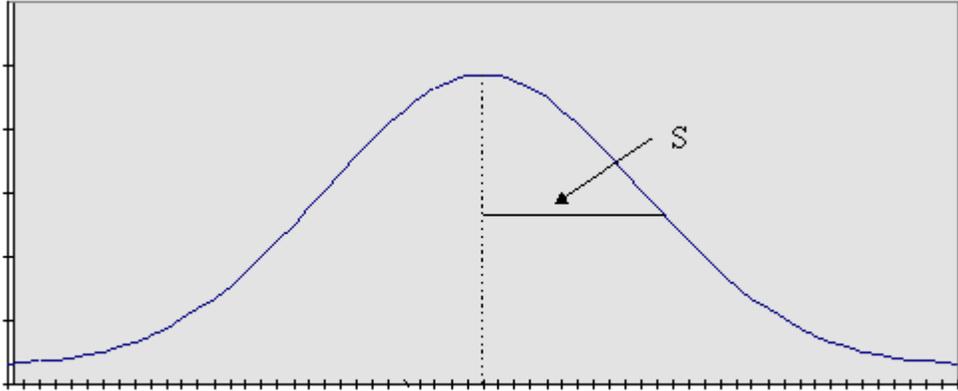
$n = 25$ vehicle maintenance records

$$\begin{aligned} S_{\bar{x}} &= \frac{4}{\sqrt{25}} \\ &= \frac{4}{5} \\ &= .8 \text{ months} \end{aligned}$$

Normal Probability Distribution . The normal probability distribution is the most commonly used continuous distribution. Because of its unique properties, it applies to many situations in which it is necessary to make inferences about a population by taking samples. It is a close approximation of the distribution of such things as human characteristics (e.g., height, weight, and intelligence) and the output of manufacturing processes (e.g., fabrication and assembly). The normal probability distribution provides the probability of a continuous random variable, and has the following characteristics:

- It is a symmetrical (i.e., the mean, median, and mode are all equal) distribution and half of the possible outcomes are on each side of the mean.
- The total area under the normal curve is equal to 1.00. In other words, there is a 100 percent probability that the possible observations drawn from the population will be covered by the normal curve.
- It is an asymptotic distribution (the tails approach the horizontal axis but never touch it).
- It is represented by a smooth, unimodal, bell-shaped curve, usually called a "normal probability density function" or "normal curve."
- It can be defined by two characteristics-the mean and the standard deviation. (See the figure below.)

NORMAL CURVE



\bar{x}

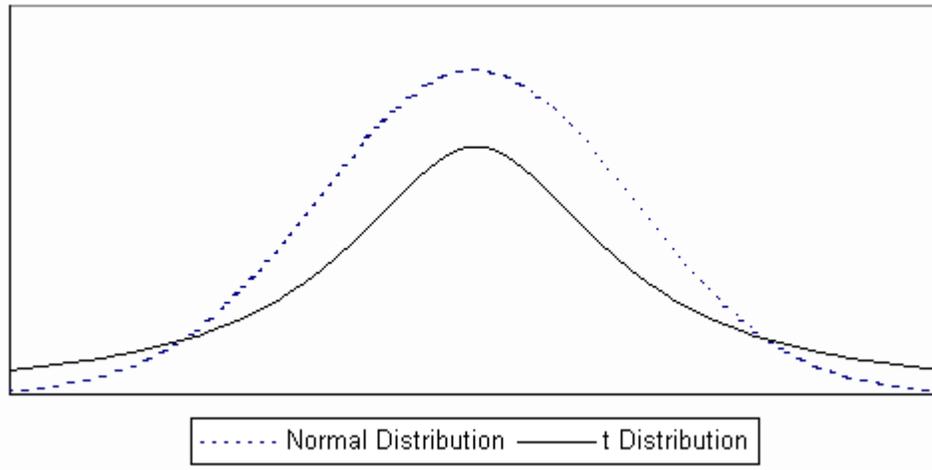
Conditions for Using the Normal Distribution . You can use the normal curve to construct confidence intervals around a sample mean when you know the population mean and standard deviation.

t Distribution . In contract pricing, the conditions for using the normal curve are rarely met. As a result, you will normally need to use a variation of the normal distribution called the "**t-distribution** ."

The t distribution has the following characteristics:

- It is symmetrical, like the normal distribution, but it is a flatter distribution (higher in the tails).
- Whereas a normal distribution is defined by the mean and the standard deviation, the t distribution is defined by degrees of freedom.
- There is a different t distribution for each sample size.
- As the sample size increases, the shape of the t distribution approaches the shape of the normal distribution.

t Distribution



Relationship Between Confidence Level and Significance Level .

Confidence Level. The term confidence level refers to the confidence that you have that a particular interval includes the population mean. In general terms, a confidence interval for a population mean when the population standard deviation is unknown and $n < 30$ can be constructed as follows:

$$\bar{x} \pm tS_{\bar{x}}$$

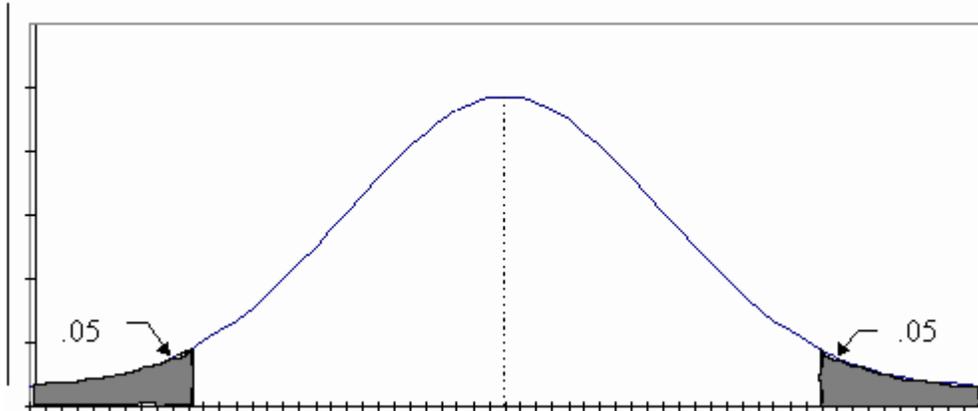
Where:

t = t Table value based on sample size and the significance level

S = Standard error of the mean

Significance Level.) The significance level is equal to 1.00 minus the confidence level. For example if the confidence level is 95 percent, the significance level is 5 percent; if the confidence level is 90 percent, the significance level is 10 percent. The significance level is, then, the area outside the interval which is likely to contain the population mean.

The figure below depicts a 90 percent confidence interval. Note that the significance level is 10 percent -- a 5 percent risk that the population mean is greater than the confidence interval plus a 5 percent risk that the mean is less than the confidence interval.



90 Percent Confidence Interval

Setting the Significance Level . When you set the significance level, you must determine the amount of risk you are willing to accept that the confidence interval does not include the true population mean. As the amount risk that you are willing to accept decreases, the confidence interval will increase. In other words, to be more sure that the true population mean is included in the interval, you must widen the interval.

Your tolerance for risk may vary from situation to situation, but for most pricing decisions, a significance level of .10 is appropriate.

Steps for Determining the Appropriate t Value for Confidence Interval Construction . After you have taken a random sample, calculated the sample mean and the standard error of the mean, you need only a value of t to construct a confidence interval. To obtain the appropriate t value, use the following steps:

Step 1. Determine your desired significance level. As stated above, for most contract pricing situations, you will find a significance level of .10 appropriate. That will provide a confidence level of .90 ($1.00 - .10 = .90$).

Step 2. Determine the degrees of freedom. Degrees of freedom (df) are the sample size minus one ($n - 1$).

Step 3. Determine the t value from the t Table . Find the t value at the intersection of the df row and the .10 column.

Constructing a Confidence Interval for the Transmission Overhaul Example . Recall the transmission overhaul example, where you wanted to estimate the useful life of transmissions of a fleet of 300 light utility trucks. We took a random sample of size $n = 25$ and calculated the following:

$$\bar{x} \pm tS_{\bar{x}}$$

Where:

$$\bar{X} = 38 \text{ months}$$

$$S = 4 \text{ months}$$

$$s_{\bar{X}} = .8 \text{ months}$$

Assume that you want to construct a 90% confidence interval for the population mean (the actual average useful life of the transmissions). You have all the values that you need to substitute into the formula for confidence interval except the t value. To determine the t value for a confidence interval, use the following steps:

Determine the appropriate t value:

Step 1. Determine the significance level. Use the significance level of .10.

Step 2. Determine the degrees of freedom.

$$\begin{aligned} df &= n - 1 \\ &= 25 - 1 \\ &= 24 \end{aligned}$$

Step 3. Determine the t value from the t Table . Find the t value at the intersection of the df = 24 row and the .10 column. The following table is an excerpt of the t Table:

Partial t Table	
df	t

23	1.714
24	1.711
25	1.708
26	1.706

Reading from the table, the appropriate t value is 1.711.

Use the t Value and Other Data to Construct Confidence Interval:

The confidence interval for the true population mean (the actual average useful life of the transmissions) would be:

$$\bar{x} \pm tS_{\bar{x}}$$

$$38.1711 (.8)$$

$$38.137 \text{ (rounded from 1.3688)}$$

Confidence interval for the population mean (): $36.63 < < 39.37$

That is, you would be 90 percent confident that the average useful life of the transmissions is between 36.63 and 39.37 months.

3.5 Using Stratified Sampling

Stratified Sampling Applications in Contract Pricing . You should consider using sampling when you have a large amount of data and limited time to conduct your analysis. While there are many different methods of sampling, stratified sampling is usually the most efficient and effective method of sampling for cost/price analysis. Using stratified sampling allows you to concentrate your efforts on the items with the greatest potential for cost/price reduction while using random sampling procedures to identify any general pattern of overpricing of smaller value items.

The most common contract pricing use of stratified sampling is analysis of detailed material cost proposals. Often hundreds, even thousands, of material items are purchased to support production of items and systems to meet Government requirements. To analyze the quantity requirements and unit prices for each item would be extremely time consuming and expensive. Effective review is essential, because often more than 50 percent of the contract price is in material items. The overall environment is custom made for the use of stratified sampling.

Steps in Stratified Sampling . In stratified sampling, the components of the proposed cost (e.g., a bill of materials) to be analyzed are divided into two or more groups or strata for analysis. One group or stratum is typically identified for 100 percent review and the remaining strata are analyzed on a sample basis. Use the following steps to develop a negotiation position based on stratified sampling:

Step 1. Identify a stratum of items that merit 100 percent analysis. Normally, these are high-value items that merit the cost of 100 percent analysis. However, this stratum may also include items identified as high-risk for other reasons (e.g., a contractor history of overpricing).

Step 2. Group the remaining items into one or more stratum for analysis. The number of additional strata necessary for analysis will depend on several factors:

- If the remaining items are relatively similar in price and other characteristics (e.g., industry, type of source, type of product), only one additional stratum may be required.
- If the remaining items are substantially different in price or other characteristics, more than one stratum may be required. For example, you might create one stratum for items with a total cost of \$5,001 to \$20,000 and another stratum for all items with a total cost of \$5,000 or less.
- If you use a sampling procedure that increases the probability of selecting larger dollar items (such as the dollar unit sampling procedure available in E-Z-Quant), the need for more than one stratum may be reduced.

Step 3. Determine the number of items to be sampled in each stratum. You must analyze all items in the strata requiring 100 percent analysis. For all other strata, you must determine how many items you will sample. You should consider several factors in determining sample size. The primary ones are variability, desired confidence, and the total count of items in the stratum. Use statistical tables or computer programs to determine the proper sample size for each stratum.

Step 4. Select items for analysis. In the strata requiring random sampling, each item in the stratum must have an equal chance of being selected and each item must only be selected once for analysis. Assign each item in the population a sequential number (e.g., 1, 2, 3; or 1001, 1002, 1003). Use a table of random numbers or computer generated random numbers to identify the item numbers to be included in the sample.

Step 5. Analyze all items identified for analysis, summing recommended costs or prices for the 100 percent analysis stratum and developing a decrement factor for any stratum being randomly sampled. In the stratum requiring 100 percent analysis, you can apply any recommended price reductions directly to the items involved. In any stratum where you use random sampling, you must apply any recommended price reductions to all items in the stratum.

- Analyze the proposed cost or price of each sampled item.
- Develop a "should pay" cost or price for the item. You must do this for every item in the sample, regardless of difficulty, to provide statistical integrity to the results. If you cannot develop a position on a sampled item because offeror data for the item is plagued by excessive misrepresentations or errors, you might have to discontinue your analysis and return the proposal to the offeror for correction and update.
- Determine the average percentage by which should pay prices for the sampled items differ from proposed prices. This percentage is the **decrement factor** .

(There are a number of techniques for determining the "average" percentage which will produce different results. For example, you could (1) determine the percentage by which each should pay price differs from each proposed price, (2) sum the percentages, and (3) divide by the total number of items in the sample. This technique gives equal weight to all sampled items in establishing the decrement factor. Or you could (1) total proposed prices for all sampled items, (2) total the dollar differences between should pay and proposed prices, and (3) divide the latter total by the former total. This technique gives more weight to the higher priced sampled items in establishing the decrement factor.)

- Calculate the confidence interval for the decrement factor.

Step 6. Apply the decrement factor to the total proposed cost of all items in the stratum .

The resulting dollar figure is your prenegotiation position for the stratum. Similarly, use confidence intervals to develop the negotiation range.

Step 7. Sum the prenegotiation positions for all strata to establish your overall position on the cost category .

Stratified Sampling Example . Assume you must analyze a cost estimate that includes 1,000 material line items with a total cost of \$2,494,500. You calculate that you must analyze a simple random sample of 50 line items.

Step 1. Identify a stratum of items that merit 100 percent analysis. You want to identify items that merit 100 percent analysis because of their relatively high cost. To do this, you prepare a list of the 1,000 line items organized from highest extended cost to lowest extended. The top six items on this list look like this:

Item 1: \$675,000

Item 2: \$546,500

Item 3: \$274,200

Item 4: \$156,800

Item 5: \$101,300

Item 6: \$ 26,705

Note that the top five items \$1,753,800 (about 70 percent of the total material cost). You will commonly find that a few items account for a large portion of proposed material cost. Also note that there is a major drop from \$101,300 to \$26,705. This is also common. Normally, you should look for such break points in planning for analysis. By analyzing Items 1 to 5, you will consider 70 percent of proposed contract cost. You can use random sampling procedures to identify pricing trends in the remaining 30 percent.

Step 2. Group the remaining items into one or more stratum for analysis. A single random sampling stratum is normally adequate unless there is a very broad range of prices requiring analysis. This typically only occurs with multimillion dollar proposals. Here, the extended prices for the items identified for random sampling range from \$5.00 to \$26,705. While this is a wide range, the dollars involved seem to indicate that a single random sampling strata will be adequate.

Step 3. Determine the number of items to be sampled in each stratum. Based on the dollars and the time available, you determine to sample a total of 20 items from the remaining 995 on the bill of materials.

Step 4. Select items for analysis.

- One way that you could select items for analysis would be putting 1,000 sheets of paper, one for each line item, into a large vat, mix them thoroughly, and select 20 slips of paper from the vat. If the slips of paper were thoroughly mixed, you would identify a simple random sample.
- A less cumbersome method would be to use a random number table (such as the example below) or a computer program to pick a simple random sample. A random number is one in which the digits 0 through 9 appear in no particular pattern and each digit has an equal probability (1/10) of occurring.
- The number of digits in each random number should be greater than or equal to the number of digits we have assigned to any element in the population.
- To sample a population of 995 items, numbered 1 to 995, random numbers must have at least three digits. Since you are dealing with three digit numbers, you only need to use the first three digits of any random number that includes four or more digits.
- Using a random number table below:
 - You could start at any point in the table. However, it is customary to select a start point at random. Assume that you start at Row 2, Column 3. The first number is 365; hence the first line item in our sample would be the item identified as 365.
 - Proceed sequentially until all 20 sample line items have been selected. The second number is 265, the third 570, etc. When you get to the end of the table you would go to Row 1, Column 4.

Random Number Table				
6698450	3756022	1379873	1091250	2464369
8230671	2261081	3651245	2403835	3796196
3307349	1606742	2651096	8733169	8020931
4557294	4487241	5709649	8443898	4186082
4867520	4593304	7859673	4715519	8672562
5256666	7532438	4932550	3189230	5767320
7629754	4443063	2645070	5402375	6320447
7388907	6027106	4214053	7360106	6648860

6976195	3827223	2887476	3195410	3200243
1120094	8270134	2226093	8064010	2436229
7471111	8911616	1633582	7834674	5557861
7569916	6576524	4641095	6893288	3204198
1317932	2313736	3428616	2105629	8731495
3440584	6406297	5135877	4804056	2250677
6472532	3716016	5304620	7202428	6889909
2676982	8524653	6426810	2026354	6881881
1662752	7385363	1674986	6997446	8676962
7408077	8443370	1586219	5255558	3478729
5564193	3848937	4385234	2467063	2012393
6419575	7393468	5890046	4375027	3094506
5382237	6976185	7170044	5523693	5298851
1407827	7079107	7446749	2960020	3833765
2090333	1717118	1766538	7504068	8285177
3264983	2462752	2857463	4812842	4262484
1574530	1746847	2933809	7563765	7835932
6645401	4831573	6157406	4779259	4099476
6934253	8266361	1628707	4227992	1857184
1063302	1395104	1288533	2096146	2439038
5189339	7209557	7587864	7348618	6628536
6771677	7664148	2353445	8520402	5754865

7643316	8106527	8565980	1717450	4791406
5672442	2860934	3900542	1489100	3935202
5152933	1853083	3480353	3585772	5243559
2493145	5267758	2810982	5584309	6599457
2270414	5399702	7469248	7473486	6377260
6916635	3169397	8863103	3877732	5318753
6543891	8863730	8075579	1949660	7169566

Step 5. Analyze all items identified for analysis summing recommended costs or prices for the 100 percent analysis stratum and developing a decrement factor for any stratum being randomly sampled.

Results of 100 Percent Analysis. Use of the 100 percent analysis is straight forward. In this example, the offeror proposed a total of \$1,753,800 for 5 line items of material. An analysis of these items found that the unit cost estimates were based on smaller quantities than required for the contract. When the full requirement was used, the total cost for those five items decreased to \$1,648,600. Since the analysis considered all items in the stratum, you simply need to use the findings in objective development.

Random Sample Results. The random sample included 20 items with an estimated cost of \$75,000. Analysis finds that the cost of the sampled items should be only 98 percent of the amount proposed. However, the confidence interval indicates that costs may range from 96 to 100 percent of the costs proposed.

Step 6. Apply the decrement factor to the total proposed cost of all items in the stratum.

Results of 100 Percent Analysis. There is no need to apply a decrement factor to these items because the recommended cost of \$1,648,600 resulted from analysis of all items in this stratum.

Random Sample Results. The assumption is that the sample is representative of the entire population. If the sample is overpriced, the entire population of is similarly overpriced. As a result the recommended cost objective would be \$725,886, or 98 percent of the proposed \$740,700. However, the confidence interval would be \$711,072 (96 percent of \$740,700) to \$740,700 (100 percent of \$740,700).

Step 7. Sum the prenegotiation positions for all strata to establish your overall position on the cost category.

Point Estimate. The total point estimate results from the 100 percent and random sample analyses would be \$2,374,486 (\$1,648,600 + \$725,886).

Confidence Interval: The confidence interval would run from \$2,359,672 (\$1,648,600 + \$711,072) to \$2,389,300 (\$1,648,600 + \$740,700). Note that the position on the stratum subject to 100 percent analysis would not change.

3.6 Identifying Issues and Concerns

Questions to Consider in Analysis . As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use statistical analysis.

- *Are the statistics representative of the current contracting situation?*

Whenever historical information is used to make an estimate of future contract performance costs, assure that the history is representative of the circumstances that the contractor will face during contract performance.

- *Have you considered the confidence interval in developing a range of reasonable costs?*

Whenever sampling procedures are used, different samples will normally result in different estimates concerning contractor costs. Assure that you consider the confidence interval in making your projections of future costs. Remember that there is a range of reasonable costs and the confidence interval will assist you in better defining that range.

- *Is the confidence interval so large as to render the point estimate meaningless as a negotiation tool?*

If the confidence interval is very large (relative to the point estimate), you should consider increasing the sample size or other means to reduce the risk involved.

- *Is your analysis, including any sample analysis, based on current, accurate, and complete information?*

A perfect analysis of information that is not current accurate and complete will likely not provide the best possible estimates of future contract costs.

- *Do the items with questioned pricing have anything in common?*

If items with questioned pricing are related, consider collecting them into a separate stratum for analysis. For example, you might find that a large number of pricing questions are related to quotes from a single subcontractor. Consider removing all items provided by that subcontractor from existing strata for separate analysis

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4.0 - Chapter Introduction

In this chapter, you will learn to use cost estimating relationships to estimate and analyze estimates of contract cost/price.

Cost Estimating Relationship Definition. As the name implies, a cost estimating relationship (CER) is a technique used to estimate a particular cost or price by using an established relationship with an independent variable. If you can identify an independent variable (driver) that demonstrates a measurable relationship with contract cost or price, you can develop a CER. That CER may be mathematically simple in nature (e.g., a simple ratio) or it may involve a complex equation.

The goal is to create a statistically valid cost estimating relationship using historical data. The parametric CER can then be used to estimate the cost of the new program by entering its specific characteristics into the parametric model. CERs established early in a program's life cycle should be continually revisited to make sure they are current and the input range still applies to the new program. In addition, parametric CERs should be well documented, because serious estimating errors could occur if the CER is improperly used.

It is important to make sure that the program attributes being estimated fall within (or, at least, not far outside) the CER dataset. For example, if a new software program was expected to contain 1 million software lines of code and the data points for a software CER were based on programs with lines of code ranging from 10,000 to 250,000, it would be inappropriate to use the CER to estimate the new program.

Among the several advantages to using cost estimating relationships are :

- **Versatility:** If the data are available, parametric relationships can be derived at any level, whether system or subsystem component. And as the design changes, CERs can be quickly modified and used to answer what-if questions about design alternatives.
- **Sensitivity:** Simply varying input parameters and recording the resulting changes in cost can produce a sensitivity analysis.
- **Statistical output:** Parametric relationships derived from statistical analysis generally have both objective measures of validity (statistical significance of each estimated coefficient and of the model as a whole) and a calculated standard error that can be used in risk analysis. This information can be used to provide a confidence level for the estimate, based on the CERs predictive capability.

- Objectivity: CERs rely on historical data that provide objective results. This increases the estimates defensibility.

Disadvantages of CERs include :

- Database requirements: The underlying database must be consistent and reliable. It may be time consuming to normalize the data or to ensure that the data were normalized correctly, especially if someone outside the estimators team developed the CER. Without understanding how the data were normalized, the analyst has to accept the database on faith-sometimes called the black-box syndrome, in which the analyst simply plugs in numbers and unquestioningly accepts the results. Using a CER in this manner can increase the estimates risk.
- Currency: CERs must represent the state of the art; that is, they must be updated to capture the most current cost, technical, and program data. Existing CERs must be validated that the current data still result in the established CER.
- Relevance: Using data outside the CER range may cause errors, because the CER loses its predictive ability for data outside the development range.
- Complexity: Complicated CERs (such as nonlinear CERs) may make it difficult for others to readily understand the relationship between cost and its independent variables.

Steps for Developing a CER . Strictly speaking, a CER is not a quantitative technique. It is a framework for using appropriate quantitative techniques to quantify a relationship between an independent variable and contract cost or price.

Developing a CER is a 6-step process. Follow the six steps whenever you develop a CER. Whenever you evaluate a CER developed by someone else, determine whether the developer followed the six steps properly.

Step 1. Define the dependent variable (e.g., cost dollars, hours, and so forth.) Define what the CER will estimate. Will the CER be used to estimate price, cost dollars, labor hours, material cost, or some other measure of cost? Will the CER be used to estimate total product cost or estimate the cost of one or more components? The better the definition of the dependent variable, the easier it will be to gather comparable data for CER development.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable. In selecting potential independent variables for CER development:

- Draw on personal experience, the experience of others, and published sources of information. When developing a CER for a new state-of-the-art item, consult experts experienced with the appropriate technology and production methods.
- Consider the following factors:
 - Variables should be quantitatively measurable. Parameters such as maintainability are difficult to use in estimating because they are difficult to measure quantitatively.
 - Data availability is also important. If you cannot obtain historical data, it will be impossible to analyze and use the variable as a predictive tool. For example, an

independent variable such as physical dimensions or parts count would be of little value during the conceptual phase of system development when the values of the independent variables are not known. Be especially wary of any CER based on 2 or 3 data observations.

- If there is a choice between developing a CER based on performance or physical characteristics, performance characteristics are generally the better choice, because performance characteristics are usually known before design characteristics.

Step 3. Collect data concerning the relationship between the dependent and independent variables. Collecting data is usually the most difficult and time-consuming element of CER development. It is essential that all data be checked and double checked to ensure that all observations are relevant, comparable, and relatively free of unusual costs.

Step 4. Explore the relationship between the dependent and independent variables. During this step, you must determine the strength of the relationship between the independent and dependent variables. This phase of CER development can involve a variety of analytical techniques from simple graphic analysis to complex mathematical analysis. Simple ratio analysis, moving averages, and linear regression are some of the more commonly used quantitative techniques used in analysis.

Step 5. Select the relationship that best predicts the dependent variable. After exploring a variety of relationships, you must select the one that can best be used in predicting the dependent variable. Normally, this will be the relationship that best predicts the values of the dependent variable. A high correlation (relationship) between a potential independent variable and the dependent variable often indicates that the independent variable will be a good predictive tool. However, you must assure that the value of the independent variable is available in order for you to make timely estimates. If it is not, you may need to consider other alternatives.

Step 6. Document your findings. CER documentation is essential to permit others involved in the estimating process to trace the steps involved in developing the relationship. Documentation should involve the independent variables tested, the data gathered, sources of data, time period of the data, and any adjustments made to the data.

4.1 - Identifying Situations For Use

Situations for Use . You can use a cost estimating relationship (CER) in any situation where you quantify one of the following:

- **A relationship between one or more product characteristics and contract cost or price.** A **product-to-cost relationship** uses product physical or performance characteristics to estimate cost or product price. The characteristic or characteristics selected for CER development are usually not the only ones driving cost, but the movement of cost has been found to be related to changes in these characteristics. The

following table identifies several product characteristic that have been used in CER development:

Product	Independent Variable
Building Construction	Floor space, roof surface area, wall surface
Gears	Net weight, gross weight, horsepower, number of driving axles, loaded cruising speed
Trucks	Empty weight, gross weight, horsepower, number of driving axles, loaded cruising speed
Passenger Car	Curb weight, wheel base, passenger space, horsepower
Turbine Engine	Dry weight, maximum thrust, cruise thrust, specific fuel consumption, by-pass ratio, inlet temperature
Reciprocating Engine	Dry weight, piston displacement, compression ratio, horsepower
Sheet Metal	Net weight, percent of scrap, number of holes drilled, number of rivets placed, inches of welding, volume of envelope
Aircraft	Empty weight, speed, useful load, wing area, power, landing speed
Diesel Locomotive	Horsepower, weight, cruising speed, maximum load on standard grade at standard speed

- A relationship between one or more elements of contract cost and another element of contract cost or price.** A **cost-to-cost relationship** uses one or more elements of contract cost to estimate cost or product price. If you can establish a relationship between different elements of cost (e.g., between senior engineering labor hours and engineering technician hours), you can use a CER to reduce your estimating or analysis effort while increasing accuracy. If you can establish a relationship between an element of cost and total price (e.g., between direct labor cost and total price), you can use that information to supplement price analysis, without requiring extensive cost information.

4.2 - Identifying And Using Rules Of Thumb

Identifying Rules of Thumb . As you perform your market analysis, you should be on the lookout for cost estimating rules of thumb that are commonly used in the product marketplace. For example, when we compare the prices of houses, we typically do so in price per square foot. Using this rule of thumb, we can compare the cost of different houses or the same house in different parts of the country. There may be ways to develop more accurate estimates, but this rule of thumb is widely accepted, relatively easy to calculate, and it provides reasonably accurate results for many purposes. The same statement can probably be made about most rules of thumb. You may be able to develop better cost estimating relationships, but given the time available and the dollars involved, rules of thumb often provide useful tools for contract pricing.

Validate a Rule of Thumb Before Using . Like any CER, a rule of thumb can be based on another cost, performance characteristic, or physical characteristic of the item being priced. Unlike other CERs, rules of thumb typically have not been validated for use in specific estimating situations. Validation has come from acceptable results produced in a variety of situations over a number of years. Before you use a rule of thumb, consider the 6-step CER development process and ask the following questions:

- ***Can the rule of thumb reasonably be used to estimate what you are trying to estimate (e.g., cost, hours, or price)?***
- ***Are there other rules of thumb that can be used to estimate the same cost, hours, or price?***
- ***Are the data required to use this rule of thumb readily available?***
- ***Does the rule of thumb provide reasonably accurate results?***
- ***If more than one rule of thumb is available, which one appears to produce the most accurate estimate?***
- ***Have technical experts or other buyers documented the results obtained from using the rule of thumb?***

Example of Rule of Thumb Validation. You just received two offers for 500 laboratory tables. Each table is 4' x 6' (24 square feet of surface area), with an oak frame and legs. The work surface is a unique composite material developed to meet Government requirements. The low offer is \$425; that offer is \$175 less than the second low offer and \$180 less than the Government estimate. As a result, you are concerned that the price may be unreasonably low. You have no acquisition history for this item and there are no similar items on the commercial market. As a result, you have been looking for a CER that you can use in your pricing decision. Another buyer, who has acquired similar tables, tells you that he has used a rule of thumb in pricing similar tables -- \$19 per square feet of surface area. You want to know the answers to the following questions before you use it in making your own pricing decisions.

- ***Can the rule of thumb reasonably be used to estimate what you are trying to estimate (e.g., cost dollars, hours, or product price)?***

The answer appears to be yes. The buyer who recommended the CER has used it successfully. Additional information shows that he learned of the CER from the scientists who developed the table-top material.

- *Are there other rules of thumb that can be used to estimate the same cost or price?*

You have asked several "experts" and have been unable to identify any other rules of thumb for estimating the price of these unique tables.

- *Are the data required to use this rule of thumb readily available?*

Yes, you already know the table surface area.

- *Does the rule of thumb provide reasonably accurate results?*

You have identified four recent acquisitions of similar tables and recorded the following information comparing the estimates made using the rule of thumb and the actual prices paid:

Sq Ft	Estimate	Actual Price	Percentage Difference
15	\$285	\$310	+ 8.8%
18	\$342	\$335	- 2.0%
32	\$608	\$580	- 4.6%

This sample size is too small to perform an effective statistical analysis, but you can still subjectively evaluate rule of thumb estimate accuracy. All estimates are within 8.8 percent of the actual price. For a rule of thumb, this appears reasonably accurate, especially since our evaluation did not consider other acquisition situation differences (e.g., the number of tables on each contract).

- *If more than one rule of thumb is available, which one appears to produce the most accurate estimates?*

In this example, there is only one known rule of thumb to consider.

- *Have technical experts or other buyers documented the results obtained from using the rule of thumb?*

In this case, the buyer documented every contract file when the rule of thumb was used. Such documentation is not only valuable in supporting the contracting officer's decision on price reasonableness; it provides valuable information to any contracting officer considering rule of thumb use in the future.

Example of Using a Rule of Thumb in Estimate Development . Once you have determined that a rule of thumb is acceptable for estimate development, you must apply it to the available data. Using this rule of thumb, your estimate would be \$456 (24 x \$19). That estimate is about 7.3 percent higher than the low offer. Based on the rule of thumb, the price does not seem unreasonable.

4.3 - Developing And Using Estimating Factors

Situations for Using Estimating Factors . An estimating rate or factor is a simple ratio, used to estimate cost or price. The rule of thumb used to develop table price estimates in the previous section is an example -- \$19 per square foot. As the size of the table top increases, the price estimate increases in direct proportion. Most rules of thumb are simple factors. Many CERs developed by Government or industry are also simple factors. They are relatively easy to develop, easy to understand, and in many cases quite accurate.

Development and use of estimating rates and factors involves two important implicit assumptions.

- There is no significant element of the cost or price being estimated that is not related to the independent variable (i.e., there is no "fixed cost" that is not associated with the independent variable).
- The relationship between the independent variable and the cost being estimated is linear.

If you believe that there are significant costs that cannot be explained by the relationship or that the relationship is not linear, you should either try to develop an equation that better tracks the true relationship or limit your use of the estimating factor to the range of the data used in developing the factor.

Example of Estimating Factor Development . Assume that you are negotiating a guard service contract for your facility and you want to develop a CER to assist you in estimating a should-pay contract price. Development should follow the 6-step CER process.

Step 1. Define the dependent variable. The objective is to develop an estimate of the price that the Government should expect to pay for this contract.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable. Logically, the major driver of price in a guard service contract is the wages paid the security guards manning the various posts identified in the contract.

Step 3. Collect data concerning the relationship between the dependent and independent variables. You have collected information on prices, minimum manning requirements, and service contract wage-rate determinations for the guard service contract at your facility for the last three years. The minimum manning requirement for the current contract totals 75,000 (Guard II) hours. The Service Contract Act (SCA) wage rate for the current year is \$10.00 per hour. The estimated direct labor cost for each year (Column D) is calculated by multiplying estimated direct labor hours (Column B) by the Service Contract Act wage rate (Column C).

A	B	C	D	E
Year	Estimated Direct	SCA Minimum	Estimated Direct	Contract Price

	Labor Hours	Wage Rate	Labor Cost	
1	87,600	\$9.15	\$801,540	\$1,346,585
2	78,840	\$9.45	\$745,038	\$1,244,215
3	70,040	\$9.50	\$665,380	\$1,124,490
Current	75,000	\$10.00	\$750,000	

Step 4. Explore the relationship between the dependent and independent variables. The following table demonstrates calculation of the Price to Direct Labor Cost Ratio. The ratio (Column F) is calculated by dividing the contract price (Column E) by the estimated direct labor cost (Column D). In Year 1 for example, price was 1.68 times the estimated direct labor cost.

A	B	C	D	E	F
Year	Estimated Direct Labor Hours	SCA Minimum Labor Rate	Estimated Direct Labor Cost	Contract Price	Price to Direct Labor Cost Ratio
1	87,600	\$9.15	\$801,540	\$1,346,585	1.68
2	78,840	\$9.45	\$745,038	\$1,244,215	1.67
3	70,040	\$9.50	\$665,380	\$1,124,490	1.69
Current	75,000	\$10.00	\$750,000		

Step 5. Select the relationship that best predicts the dependent variable. It appears from the information above, that there is a relationship between contract price and the estimated direct labor cost. The price is between 1.67 and 1.69 times the estimated direct labor cost. The average ratio is 1.68.

$$\text{Average Ratio} = (1.68 + 1.67 + 1.69) / 3 = 1.68$$

You can now use this ratio to estimate the price of similar contracts.

Step 6. Document your findings. Your documentation of CER development should include the information from the 6-step process above. Exact documentation requirements will vary with the analysis involved.

Using an Estimating Factor in Estimated Development . Once you calculate an estimating factor, you can use it to estimate should-pay price for similar products. Using the 1.68 factor from the guard contract example, you can calculate a should-pay price for the current year. Using this factor, the best estimate of a reasonable price would be \$1,260,000, as shown in the table below:

A	B	C	D	E	F
Year	Estimated Direct Labor Hours	SCA Minimum Labor Rate	Estimated Direct Labor Cost	Contract Price	Price to Direct Labor Cost Ratio
Current	75,000	\$10.00	\$750,000	\$1,260,000	1.68

Given the data above, you should be reasonably confident of your estimate, because the range of ratios is only from 1.67 to 1.69. Even without statistical analysis, that range might be useful in establishing a range of reasonable prices.

High side: $1.69 \times \$750,000 = \$1,267,500$

Mean: $1.68 \times \$750,000 = \$1,260,000$

Low side: $1.67 \times \$750,000 = \$1,252,500$

4.4 - Developing And Using Estimating Equations

Situations for Using Estimating Equations . Not all estimating relationships lend themselves to the use of simple estimating factors. If there is a substantial element of the cost or price being estimated that is not related to the independent variable (i.e., there is a "fixed cost" that is not associated with the independent variable), you should consider using a linear estimating equation. If the relationship is not linear, consider a nonlinear estimating equation.

Example of Estimating Equation Development . Assume that you are analyzing the costs proposed for the construction of a new house and decide to develop a CER to support your analysis. Development should follow the 6-step CER process described in the chapter Introduction

Step 1. Define the dependent variable. The objective is to estimate the cost of a new base housing model.

Step 2. Select independent variables to be tested for developing estimates of the dependent variable. A variety of house characteristics could be used to estimate cost. These include such characteristics as square feet of living area, exterior wall surface area, number of baths, and others.

Step 3. Collect data concerning the relationship between the dependent and independent variables.

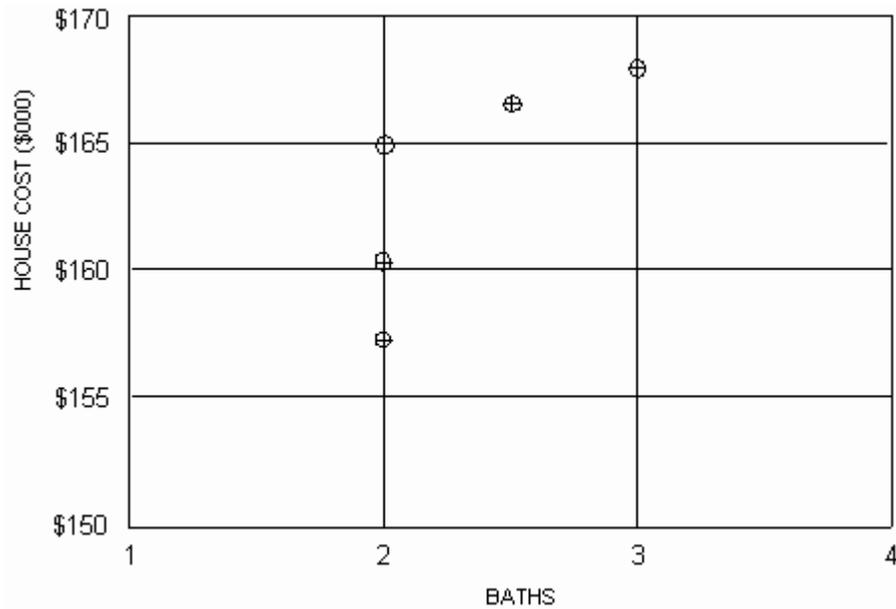
Base Housing Model	Unit Cost	Baths	Sq. Ft. Living Area	Sq. Ft. Exterior Wall
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				Surface
Burger	\$166,500	2.5	2,800	2,170
Metro	\$165,000	2.0	2,700	2,250
Suburban	\$168,000	3.0	2,860	2,190
Executive	\$160,500	2.0	2,440	1,990
Ambassador	\$157,000	2.0	1,600	1,400
New Home		2.5	2,600	2,100

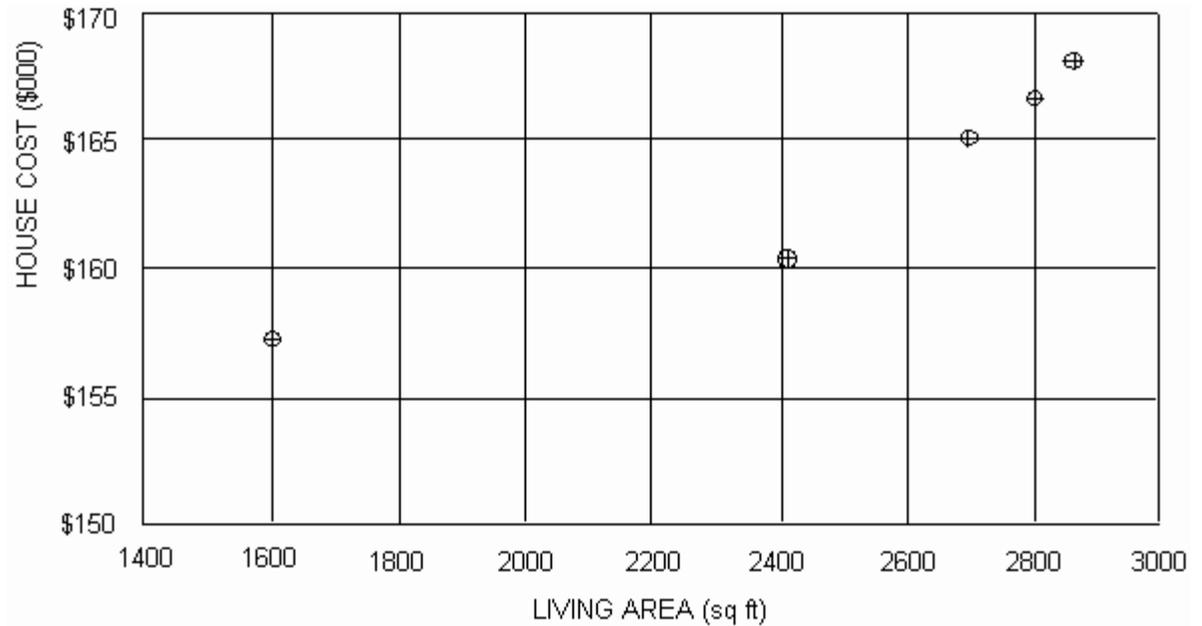
Step 4. Explore the relationship between the dependent and independent variables.

Analysis of the relationship between the independent variable and house price could be performed using many different techniques. In this situation most analysts would use regression analysis. However, here we will use graphic analysis to demonstrate the thought process involved. Three independent variables will be tested: number of baths, living area, and exterior wall surface area.

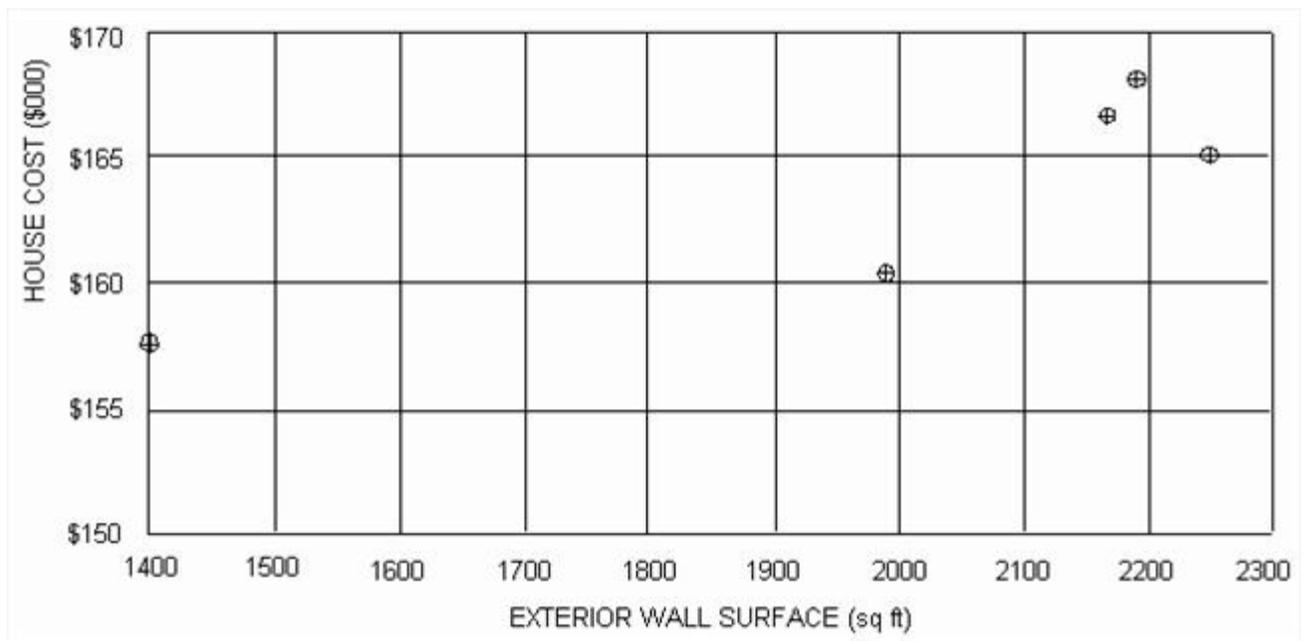
- **Price and the Number Of Baths.** This graph appears to demonstrate that the number of baths is not a good estimating tool, because three houses with a nearly \$8,000 price difference have the same number of baths.



- **Price and Square Feet Of Living Area.** This graph appears to depict a relationship.

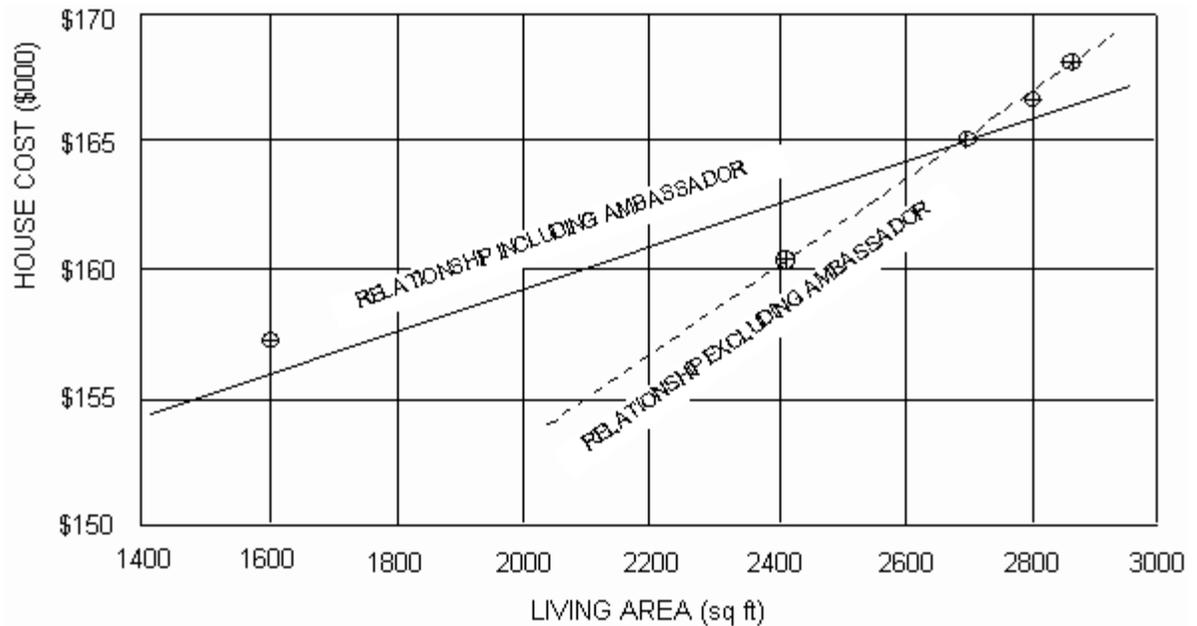


- **Price and Exterior Wall Surface Area.** This graph also appears to depict a relationship.

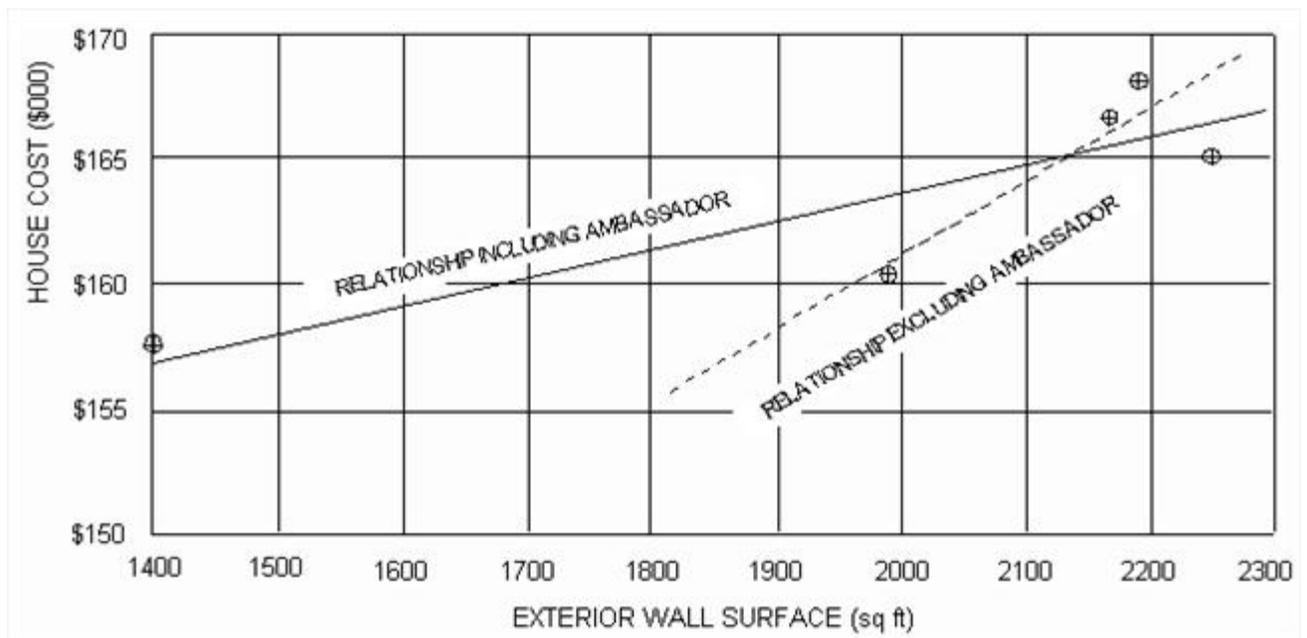


Step 5. Select the relationship that best predicts the dependent variable. Based on the initial analysis, it appears that square feet of living area and exterior wall surface have the most potential for development of a CER. The two graphs below depict efforts to fit a straight line through the observed values. Note that both graphs demonstrate efforts to fit a line with and without using the data from the Ambassador model.

- **Price and Living Area.**



- **Price and Exterior Wall Surface Area.**



Consider Analysis Results and Other Data. Viewing both of these relationships, we might question whether the Ambassador model data should be included in developing our CER. In developing a CER, you need not use all available data if all data is not comparable. However, you should not eliminate data just to get a better looking relationship. After further analysis, we find that the Ambassador's size is substantially different from the other houses for which we have

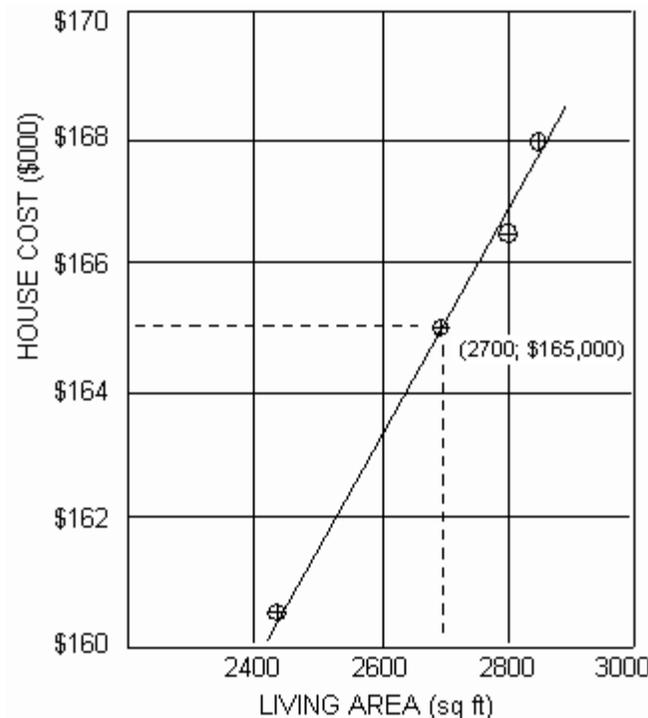
data and the house for which we are estimating. This substantial difference in size might logically affect relative construction cost. Based on this information, you might decide not to consider the Ambassador data in CER development.

Final Analysis. If you exclude the Ambassador data, you find that the fit of a straight-line relationship of cost to the exterior wall surface is improved. The relationship between cost and square feet of living area is even closer, almost a straight line.

If you had to choose one relationship, you would probably select living area over exterior wall surface because living area has so much less variance from the trend line. Since the relationship between living area and price is so close, we can reasonably use it for our CER.

If the analysis of these relationships did not reveal a useful predictive relationship, you might consider combining two or more of the relationships already explored or exploring new relationships.

Step 6. Document your findings. In documenting our findings, we can relate the process involved in selecting living area for price estimation. We may then present the following graph developed as an estimating tool.



We might also convert the graphic relationship to an equation. The cost estimating relationship (CER) would be:

$$Y = \$117,750 + (\$17.50 \times \text{Sq Ft of Living Area})$$

Using an Estimating Equation to Estimate Cost. Once developed, you can use an estimating equation to contract cost or price in similar circumstances.

For example : Applying our new CER to the estimation of cost for our new 2,600 square-foot house, we would estimate:

$$\begin{aligned} Y &= \$117,750 + (\$17.50 \times \text{Sq Ft of Living Area}) \\ &= \$117,750 + (\$17.50 \times 2,600) \\ &= \$117,750 + \$45,500 \\ &= \$163,250 \text{ estimated price} \end{aligned}$$

CERs, like most other tools of cost analysis, **MUST** be used with judgment. Judgment is required to evaluate the historical relationships in the light of new technology, new design, and other similar factors. Therefore, knowledge of the factors involved in CER development is essential to proper application of the CER.

4.5 Identifying Issues And Concerns

Questions to Consider in Analysis . As you perform price or cost analysis, consider the issues and concerns identified in this section as you consider use of a cost estimating relationship.

- ***Does the available information verify the existence and accuracy of the proposed relationship?***

Technical personnel can be helpful in analyzing the technical validity of the relationship. Audit personnel can be helpful in verifying the accuracy of any contractor data and analysis.

- ***Is there a trend in the relationship?***

For example, the cost of rework is commonly estimated as a factor of production labor. As production continues, the production effort should become more efficient and produce fewer defective units which require repair. The factor should decrease over time. You should also consider the following related questions: Is the rate distorted by one bad run? What is being done to control the rate? What else can be done?

- ***Is the CER used consistently?***

If an offeror uses a CER to propose an element of cost, it should be used in all similar proposals. Since the CER can be used to estimate the average value, some jobs should be expected to cost more and others less. With a valid CER, you assume the variances will be minor and will average out across all contracts. To use a CER in some cases and a discrete estimate in others destroys its usefulness by over or understating costs across all proposals (e.g., using the average unless a discrete estimate is lower/higher negates the averaging out of cost across all contracts and is clearly unfair to one of the contracting parties).

- ***Has the CER been consistently accurate in the past?***

No matter how extensive the price/cost information or how sophisticated the analysis technique, if a CER does not do a good job of accurately projecting cost, then it is not a useful tool.

- ***How current is the CER?***

Even the most accurate CER needs to be reviewed and updated. While the time interval between updates will differ with CER sensitivity to change, in general a CER should be reviewed and updated at least annually. A CER based on a moving average should be updated whenever new data become available.

- ***Would another independent variable be better for developing and applying a CER?***

If another independent variable would consistently provide a more accurate estimate, then it should be considered. However, remember that the CER may be applicable to other proposals, not just yours. It is possible that a relationship which works well on your contract would not work well across the entire contract population. When assessing CER validity, you should consider all affected contracts.

- ***Is the CER a self-fulfilling prophecy?***

A CER is intended to project future cost. If the CER simply "backs into" a rate that will spread the cost of the existing capacity across the affected contracts, then the CER is not fulfilling its principle function. If you suspect that a CER is being misused as a method of carrying existing resources, you should consider a should-cost type review on the functions represented by the CER.

- ***Would use of a detailed estimate or direct comparison with actual cost from a prior effort produce more accurate results?***

Development of a detailed estimate can be time consuming and costly but the application of the engineering principles required is particularly valuable in estimating cost of efficient and effective contract performance.

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- 5.3 - [Analyzing Variation In The Regression Model](#)
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5.0 - Chapter Introduction

In this chapter, you will learn to use regression analysis in developing cost estimating relationships and other analyses based on a straight-line relationship even when the data points do not fall on a straight line.

Line-of-Best-Fit . The straight-line is one of the most commonly used and most valuable tools in both price and cost analysis. It is primarily used to develop cost estimating relationships and to project economic trends. Unfortunately, in contract pricing the data points that are used in analysis do not usually fall exactly on a straight line. Much of the variation in a dependent variable may be explained by a linear relationship with an independent variable, but there are usually random variations that cannot be explained by the line. The goal in establishing a line-of-best-fit is to develop a predictive relationship that minimizes the random variations. This can be done visually with a graph and a ruler, but the visual line-of-best-fit is an inexact technique and has limited value in cost or price analysis. Regression analysis is commonly used to analyze more complex relationships and provide more accurate results.

This chapter will focus on simple regression (2-variable linear regression); in which a single independent variable (X) is used to predict the value of a single dependent variable (Y). The dependent variable will normally be either price or cost (e.g., dollars or labor hours), the independent variable will be a measure related to the product (supply or service) being acquired. It may be a physical characteristic of the product, a performance characteristic of the product, or an element of cost to provide the product.

In some situations, you may need regression analysis tools that are more powerful than simple regression. Multiple regression (multivariate linear regression) and curvilinear regression are variations of simple regression that you may find useful. The general characteristics of both will be addressed later in the chapter.

5.1 - Identifying Situations For Use

Cost Estimating Relationship Development and Analysis . Regression analysis is one of the techniques most commonly used to establish cost estimating relationships (CERs) between independent variables and cost or price. If you can use regression analysis to quantify a CER, you can then use that CER to develop and analyze estimates of product cost or price.

Indirect Cost Rate Analysis ([FAR 31.203](#)). Indirect costs are costs that are not directly identified with a single final cost objective (e.g., a contract), i.e., indirect costs are identified with two or more final cost objectives or an intermediate cost objective(s). Minor direct costs may be treated as indirect costs if the treatment is consistently applied to all final cost objectives and the allocation produces substantially the same results as treating the cost as a direct cost.

[FAR 31.203](#) requires that indirect costs be accumulated into logical cost pools and allocated to final cost objectives on the basis of the benefits accruing to the various cost objectives.

Regression analysis is commonly used to quantify the relationship between the indirect cost allocation base and expense pool over time. If you can quantify the relationship, you can then use that relationship to develop or analyze indirect cost rate estimates.

Time-Series Analysis . You can use regression analysis to analyze trends that appear to be related to time. It is particularly useful when you can identify and adjust for other factors that affect costs or prices (e.g., quantity changes) to isolate the effect of inflation/deflation for analysis. The most common applications of this type are forecasting future wage rates, material costs, and product prices.

In time-series analysis, cost or price data are collected over time for analysis. An estimating equation is developed using time as the independent variable. The time periods are normally weeks, months, quarters, or years. Each time period is assigned a number (e.g., the first month is 1, the fourth month is 4, etc.). All time periods during the analysis must be considered, whether or not data were collected during that period.

Time does not cause costs or prices to change. Changes are caused by a variety of economic factors. Do not use time-series analysis when you can identify and effectively measure the factors that are driving costs or prices. If you can identify and measure one or more key factors, you should be able to develop a better predictive model than by simply analyzing cost or price changes over time. However, if you cannot practically identify or measure such factors, you can often make useful predictions by using regression analysis to analyze cost or price trends over time.

Just remember that regression analysis will not automatically identify changes in a trend (i.e., it cannot predict a period of price deflation when the available data trace a trend of increasing prices). As a result, regression analysis is particularly useful in short-term analysis. The further you predict into the future, the greater the risk.

5.2 - Developing And Using A Simple Regression Equation

Simple Regression Model . The simple regression model is based on the equation for a straight line:

$$Y_c = A + BX$$

Where:

Y_c = The calculated or estimated value for the dependent (response) variable

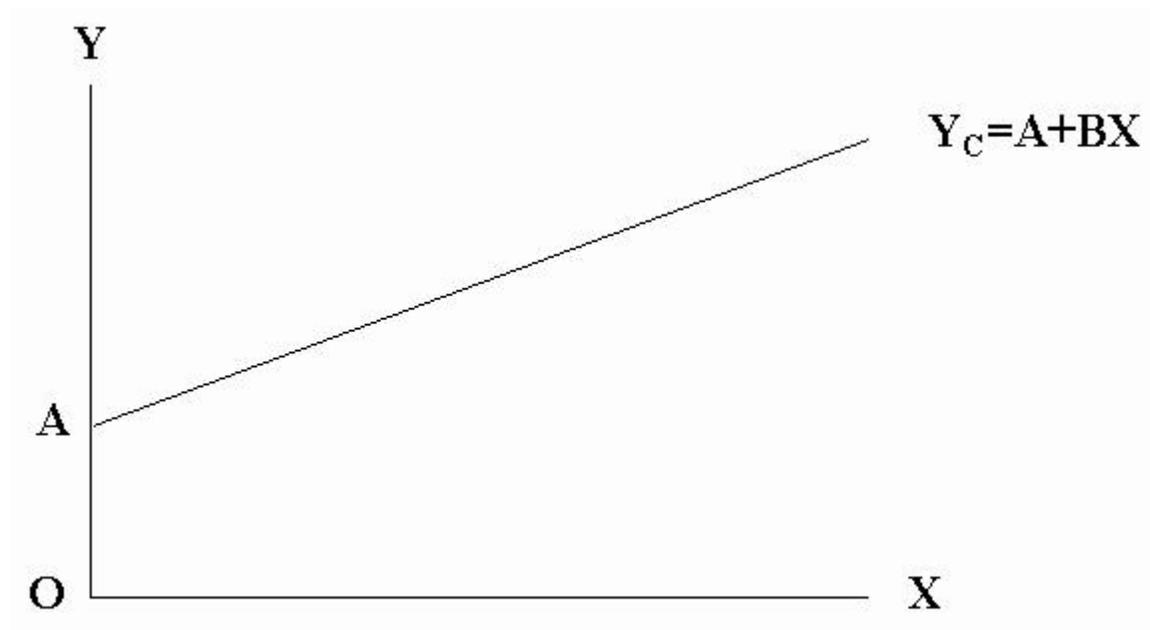
A = The Y intercept, the theoretical value of Y when $X = 0$

X = The independent (explanatory) variable

B = The slope of the line (the change in Y divided by the change in X , i.e., the value by which Y changes when X changes by one).

For a given data set, A and B are constants. They do not change as the value of the independent variable changes. Y_c is a function of X . Specifically, the functional relationship between Y_c and X is that Y_c is equal to A plus the product of B times X .

The following figure graphically depicts the regression line:



Steps for Developing a 2-Variable Linear Regression Equation . To develop a regression equation for a particular set of data, use the following 5-step least-squares-best-fit (LSBF) process:

Step 1. Collect the historical data required for analysis. Identify the X and Y values for each observation.

X = Independent variable

Y = Dependent variable

Step 2. Put the data in tabular form.

Step 3. Compute \bar{X} and \bar{Y} .

$$\bar{X} = \frac{\sum X}{n} \quad \bar{Y} = \frac{\sum Y}{n}$$

Where:

\bar{X} = Sample mean for observations the independent variable

\bar{Y} = Sample mean for observations the dependent variable

= Summation of all the variables that follow the symbol (e.g., ΣX represents the sum of all X values)

X = Observation value for the independent variable

Y = Observation value for the dependent variable

n = Total number of observations in the sample

Step 4. Compute the slope (B) and the Y intercept (A).

$$B = \frac{\Sigma XY - n \bar{X} \bar{Y}}{\Sigma X^2 - n \bar{X}^2}$$

$$A = \bar{Y} - B \bar{X}$$

Step 5. Formulate the estimating equation.

$$Y_c = A + BX$$

2-Variable Linear Regression Equation Development Example . Assume a relationship between a firm's direct labor hours and manufacturing overhead cost based on the use of direct labor hours as the allocation base for manufacturing overhead. Develop an estimating equation using direct labor hours as the independent variable and manufacturing overhead cost as the dependent variable. Estimate the indirect cost pool assuming that 2,100 manufacturing direct labor hours will be needed to meet 20X8 production requirements.

Step 1. Collect the Historical Data Required for Analysis.

Historical Data		
Year	Manufacturing Direct Labor Hours	Manufacturing Overhead
20X2	1,200	\$ 73,000
20X3	1,500	\$ 97,000
20X4	2,300	\$128,000
20X5	2,700	\$155,000

20X6	3,300	\$175,000
20X7	3,400	\$218,000
20X8	2,100 (Est)	

Step 2. Put The Data In Tabular Form.

X = Manufacturing direct labor hours in hundreds of hours (00s)

Y = Manufacturing overhead in thousands of dollars (\$000s)

	Tabular Presentation				
	X	Y	XY	X²	Y²
	12	73	876	144	5,329
	15	97	1,455	225	9,409
	23	128	2,944	529	16,384
	27	155	4,185	729	24,025
	33	175	5,775	1,089	30,625
	34	218	7,412	1,156	47,524
Column Totals	144	846	22,647	3,872	133,296

Step 3. Compute \bar{X} and \bar{Y} .

$$\begin{aligned} \bar{X} &= \frac{\Sigma X}{n} & \bar{Y} &= \frac{\Sigma Y}{n} \\ &= \frac{144}{6} & &= \frac{846}{6} \\ &= 24 & &= 141 \end{aligned}$$

Step 4. Compute the slope (B) and the intercept (A).

$$\begin{aligned}
 B &= \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2} \\
 &= \frac{22,647 - 6(24)(141)}{3,872 - 6(24)^2} \\
 &= \frac{22,647 - 20,304}{3,872 - 3,456} \\
 &= \frac{2,343}{416} \\
 &= 5.6322
 \end{aligned}$$

$$\begin{aligned}
 A &= \bar{Y} - B\bar{X} \\
 &= 141 - 5.6322(24) \\
 &= 141 - 135.1728 \\
 &= 5.8272
 \end{aligned}$$

Step 5. Formulate the estimating equation. Substitute the calculated values for A and B into the equation:

$$Y_C = A + BX$$

$$Y_C = 5.8272 + 5.6322X$$

Where:

Y_C = Manufacturing overhead (\$000's)

X = Manufacturing direct labor hours (00's)

Example of Estimate Using Simple Regression Equation . Estimate manufacturing overhead given an estimate for manufacturing direct labor hours of 2,100:

$$Y_C = 5.8272 + 5.622X$$

$$= 5.8272 + 5.622(21)$$

$$= 5.8272 + 118.2762$$

$$= 124.1034 \text{ thousand dollars}$$

Rounded to the nearest dollar, the estimate would be \$124,103.

5.3 - Analyzing Variation In The Regression Model

Assumptions of the Regression Model. The assumptions listed below enable us to calculate unbiased estimators of the population) and to use these in predicting values and regression function coefficients (of Y given X). You should be aware of the fact that violation of one or more of these assumptions reduces the efficiency of the medle, but a detailed discussion of this topic is beyond the purview of this text. Assume that all these assumptions have been met.

- For each value of X there is an array of possible Y normally distributed about the regression line.
- The mean of the distribution of possible Y values is on the regression line, i.e., the expected value of the error term is zero.
- The standard deviation of the distribution of possible Y values is constant regardless of the value of X (this is called homoscedasticity).
- The error terms are statistically independent of each other, i.e., there is no serial correlation.
- The error term is statistically independent of X.

Note: These assumptions are very important, in that they enable us to construct predictions around our point estimate.

Variation in the Regression Model . Recall that the purpose of regression analysis is to predict the value of a dependent variable given the value of the independent variable. The LSBF technique yields the best single line to fit the data, but you also want some method of determining how good this estimating equation is. In order to do this, you must first partition the variation.

- **Total Variation.** The sum of squares total (SST) is a measure of the total variation of Y. SST is the sum of the squared differences between the observed values of Y and the mean of Y.

$$SST = (Y_i - \bar{Y})^2$$

Where:

SST = Sum of squared differences

Y_i = Observed value i

\bar{Y} = Mean value of Y

While the above formula provides a clear picture of the meaning of SST, you can use the following formula to speed SST calculation:

$$SST = \Sigma Y^2 - \bar{Y} \Sigma Y$$

Total variation can be partitioned into two variations categories: explained and unexplained. This can be expressed as

$$SST = SSR + SSE$$

- **Explained Variation.** The sum of squares regression (SSR) is a measure of variation of Y that is explained by the regression equation. SSR is the sum of the squared differences between the calculated value of Y (Y_c) and the mean of Y (\bar{Y}).

$$SSR = \sum(Y_c - \bar{Y})^2$$

You can use the following formula to speed SSR calculation:

$$SSR = B(\sum XY - \bar{X}\sum Y)$$

- **Unexplained Variation .** The sum of squares error (SSE) is a measure of the variation of Y that is not explained by the regression equations. SSE is the sum of the squared differences between the observed values of Y and the calculated value of Y. This is the random variation of the observations around the regression line.

$$SSE = \sum(Y_i - Y_c)^2$$

You can use the following formula to speed SSE calculation:

$$SSE = \sum Y^2 - A\sum Y - B\sum XY$$

Analysis of Variance . Variance is equal to variation divided by degrees of freedom (df). In regression analysis, df is a statistical concept that is used to adjust for sample bias in estimating the population mean.

- **Mean Square Regression (MSR).**

$$MSR = \frac{SSR}{df}$$

For 2-variable linear regression, the value of df for calculating MSR is always one (1). As a result, in 2-variable linear regression, you can simplify the equation for MSR to read:

$$MSR = \frac{SSR}{1} \quad \text{or}$$
$$MSR = SSR$$

- **Mean Square Error (MSE).**

$$MSE = \frac{SSE}{df}$$

In 2-variable linear regression, df for calculating MSE is always $n - 2$. As a result, in simple regression, you can simplify the equation for MSE to read:

$$MSE = \frac{SSE}{n - 2}$$

- **Analysis of Variance Table.** The terms used to analyze variation/variance in the regression model are commonly summarized in an Analysis of Variance (ANOVA) table.

ANOVA Table			
Source	Sum of Squares	df	Mean Square**
Regression	SSR	1	MSR
Error	SSE	$n-2$	MSE
Total	SST	$n-1$	

**Mean Square = Sum of Squares/df

Constructing an ANOVA Table for the Manufacturing Overhead Example . Before you can calculate variance and variation, you must use the observations to calculate the statistics in the table below. Since we already calculated these statistics to develop the regression equation to estimate manufacturing overhead, we will begin our calculations with the values in the table below:

Statistic	Value
?	144
?Y	846
?XY	22,647
?X ²	3,872
?Y ²	133,296
\bar{X}	24
\bar{Y}	141
A	5.8272
B	5.6322

n	6
---	---

Step 1. Calculate SST.

$$\begin{aligned}
 SST &= \sum Y^2 - \bar{Y}\sum Y \\
 &= 133,296 - 141(846) \\
 &= 133,296 - 119,286 \\
 &= 14,010
 \end{aligned}$$

Step 2. Calculate SSR.

$$\begin{aligned}
 SSR &= B(\sum XY - \bar{X}\sum Y) \\
 &= 5.6322 [22,647 - 24(846)] \\
 &= 5.622 [22,647 - 20,304] \\
 &= 5.6322 [2,343] \\
 &= 13,196.24 \text{ (rounded to 13,196 for this example)}
 \end{aligned}$$

Step 3. Calculate SSE.

$$\begin{aligned}
 SSE &= \sum Y^2 - A\sum Y - B\sum XY \\
 &= 133,296 - 5.8272(846) - 5.6322(22,647) \\
 &= 133,296 - 4929.81 - 127,552.43 \\
 &= 813.76 \text{ (rounded to 814 for this example)}
 \end{aligned}$$

Step 4. Calculate MSR.

$$\begin{aligned}
 MSR &= SSR \\
 &= 13,196
 \end{aligned}$$

Step 5. Calculate MSE.

$$\begin{aligned}
 MSE &= \frac{SSE}{n - 2} \\
 &= \frac{814}{6 - 2} \\
 &= \frac{814}{4} \\
 &= 203.5 \text{ (rounded to 204 for this example)}
 \end{aligned}$$

Step 6. Combine the calculated values into an ANOVA table.

ANOVA Table

Source	Sum of Squares	df	Mean Square**
Regression	13,196	1	13,196
Error	814	4	204
Total	14,010	5	
**Mean Square = Sum of Squares/df			

Step 7. Check SST. Assure that value for SST is equal to SSR plus SSE.

$$SST = SSR + SSE$$

$$14,010 = 13,196 + 814$$

$$14,010 = 14,010$$

5.4 - Measuring How Well The Regression Equation Fits The Data

Statistics Used to Measure Goodness of Fit . How well does the equation fit the data used in developing the equation? Three statistics are commonly used to determine the "goodness of fit" of the regression equation:

- Coefficient of determination;
- Standard error of the estimate; and
- T-test for significance of the regression equation.

Calculating the Coefficient of Determination . Most computer software designed to fit a line using regression analysis will also provide the coefficient of determination for that line. The coefficient of determination (r^2) measures the strength of the association between independent and dependent variables (X and Y).

The range of r^2 is between zero and one.

$$0 < r^2 < 1$$

An r^2 of zero indicates that there is no relationship between X and Y. An r^2 of one indicates that there is a perfect relationship between X and Y. As r^2 gets closer to 1, the better the regression line fits the data set.

In fact, r^2 is the ratio of explained variation (SSR) to total variation (SST). An r^2 of .90 indicates that 90 percent of the variation in Y has been explained by its relationship with X; that is, 90 percent of the variation in Y has been explained by the regression line.

$$r^2 = \frac{SSR}{SST}$$

For the manufacturing overhead example:

$$\begin{aligned} r^2 &= \frac{13,196}{14,010} \\ &= .94 \end{aligned}$$

This means that approximately 94% of the variation in manufacturing overhead (Y) can be explained by its relationship with manufacturing direct labor hours (X).

Standard Error of the Estimate . The standard error of the estimate (SEE) is a measure of the accuracy of the estimating (regression) equation. The SEE indicates the variability of the observed (actual) points around the regression line (predicted points). That is, it measures the extent to which the observed values (Y_i) differ from their calculated values (Y_c). Given the first two assumptions required for use of the regression model (for each value of X there is an array of possible Y values which is normally distributed about the regression line and the mean of this distribution (Y_c) is on the regression line), the SEE is interpreted in a way similar to the way in which the standard deviation is interpreted. That is, given a value for X, we would generally expect the following intervals (based on the Empirical Rule):

- Y_c = 1 SEE contains approximately 68 percent of the total observations (Y_i)
- Y_c = 2 SEE contains approximately 95 percent of the total observations (Y_i)
- Y_c = 3 SEE contains approximately 99 percent of the total observations (Y_i)

The SEE is equal to the square root of the MSE.

$$SEE = \sqrt{MSE}$$

For the manufacturing overhead example:

$$\begin{aligned} SEE &= \sqrt{204} \\ &= 14.28 \end{aligned}$$

Steps for Conducting the T-test for the Significance of the Regression Equation . The regression line is derived from a sample. Because of sampling error, it is possible to get a regression relationship with a rather high r^2 (e.g. greater than 80 percent) when there is no real relationship between X and Y. That is, when there is no statistical significance. This phenomenon will occur only when you have very small sample data sets. You can test the significance of the regression equation by applying the T-test. Applying the T-test is a 4-step process:

Step 1. Determine the significance level (α).

= 1 - confidence level

The selection of the significance level is a management decision; that is, management decides the level of risk associated with an estimate which it will accept. In the absence of any other guidance, use a significance level of .10.

Step 2. Calculate T. Use the values of MSR and MSE from the ANOVA table:

$$T = \sqrt{\frac{MSR}{MSE}}$$

Step 3. Determine the table value of t. From a t Table, select the t value for the appropriate degrees of freedom (df). In 2-variable linear regression:

$$df = n - 2$$

Step 4. Compare T to the t Table value. Decision rules:

If $T > t$, use the regression equation for prediction purposes. It is likely that the relationship is significant.

If $T < t$, do not use the regression equation for prediction purposes. It is likely that the relationship is not significant.

If $T = t$, a highly unlikely situation, you are theoretically indifferent and may elect to use or not use the regression equation for prediction purposes.

Conducting the T-test for the Significance of the Regression Equation for the Manufacturing Overhead Example .

To demonstrate use of the T-test, we will apply the 4-step procedure to the manufacturing overhead example:

Step 1. Determine the significance level (). Assume that we have been told to use = .05.

Step 2. Calculate T.

$$\begin{aligned} T &= \sqrt{\frac{MSR}{MSE}} \\ &= \sqrt{\frac{13,196}{204}} \\ &= \sqrt{64.69} \\ &= 8.043 \end{aligned}$$

Step 3. Determine the table value of t. The partial table below is an excerpt of a t table.

$$\begin{aligned}
 df &= n - 2 \\
 &= 6 - 2 \\
 &= 4
 \end{aligned}$$

Partial t Table	
df	t

2	4.303
3	3.182
4	2.776
5	2.571
6	2.447

Reading from the table, the appropriate value is 2.776.

Step 4. Compare T to the t Table value. Since $T (8.043) > t (2.776)$, use the regression equation for prediction purposes. It is likely that the relationship is significant.

Note: There is not normally a conflict in the decision indicated by the T-test and the magnitude of r^2 . If r^2 is high, T is normally $> t$. A conflict could occur only in a situation where there are very few data points. In those rare instances where there is a conflict, you should accept the decision indicated by the T-test. It is a better indicator than r^2 because it takes into account the sample size (n) through the degrees of freedom (df).

5.5 - Calculating And Using A Prediction Interval

Formulating the Prediction Interval. You can develop a regression equation and use it to calculate a point estimate for Y given any value of X. However, a point estimate alone does not provide enough information for sound negotiations. You need to be able to establish a range of values which you are confident contains the true value of the cost or price which you are trying to predict. In regression analysis, this range is known as the prediction interval.

For a regression equation based on a small sample, you should develop a prediction interval, using the following equation:

$$Y_c \pm t(\text{SEE}) \sqrt{1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{\sum X^2 - n\bar{X}^2}}$$

$$124.1034 \pm 2.776(14.27) \sqrt{1 + \frac{1}{6} + \frac{(21 - 24)^2}{3,872 - 6(24)^2}}$$

$$124.1034 \pm 39.6135 \sqrt{1 + .1667 + \frac{(-3)^2}{3,872 - 3,456}}$$

$$124.1034 \pm 39.6135 \sqrt{1.1667 + \frac{9}{416}}$$

$$124.1034 \pm 39.6135 \sqrt{1.1667 + .0216}$$

$$124.1034 \pm 39.6135 \sqrt{1.1883}$$

$$124.1034 \pm 39.6135 (1.0901)$$

$$124.1034 \pm 43.1827$$

When $X = 21$ the prediction interval is 80.9207 Y 167.2861

Constructing a Prediction Interval for the Manufacturing Overhead Example . Assume that we want to construct a 95 percent prediction interval for the manufacturing overhead estimate at 2,100 manufacturing direct labor hours. Earlier in the chapter, we calculated Y_c and the other statistics in the following table:

Statistic	Value
Y_c	124.1034
t (Use $n - 2$ df)	2.776
SEE	14.27
\bar{X}	24
$\sum X^2$	3,872

Using the table data, you would calculate the prediction interval as follows: When $X = 21$ the prediction interval is: 80.9207 Y 167.2861.

Prediction Statement: We would be 95 percent confident that the actual manufacturing overhead will be between \$80,921 and \$167,286 at 2,100 manufacturing direct labor hours.

5.6 - Identifying The Need For Advanced Regression Analysis

Other Forms of Regression . In 2-variable regression analysis, you use a single independent variable (X) to estimate the dependent variable (Y), and the relationship is assumed to form a straight line. This is the most common form of regression analysis used in contract pricing. However, when you need more than one independent variable to estimate cost or price, you should consider multiple regression (or multivariate linear regression). When you expect that a trend line will be a curve instead of a straight line, you should consider curvilinear regression.

A detailed presentation on how to use multiple regression or curvilinear regression is beyond the scope of this text. However, you should have a general understanding of when and how these techniques can be applied to contract pricing. When you identify a situation that seems to call for the use of one of these techniques, consult an expert for the actual analysis. You can obtain more details on the actual use of these techniques from advanced forecasting texts.

Multiple Regression Situation . Multiple regression analysis assumes that the change in Y can be better explained by using more than one independent variable. For example, suppose you want to determine the relationship between main-frame computer hours, field-audit hours expended in audit analysis, and the cost reduction recommendations sustained during contract negotiations.

Computer Hours	Field Audit Hours	Sustained Reduction
1.4	45	\$290,000
1.1	37	\$240,000
1.4	44	\$270,000
1.1	45	\$250,000
1.3	40	\$260,000
1.5	46	\$280,000
1.5	47	\$300,000

It is beyond the purpose of this text to demonstrate how a multivariate equation would be developed using this data. However, we will describe the elements of the multivariate equation and the results of a regression analysis.

Three-Variable Linear Equation . Multiple regression can involve any number of independent variables. To solve the audit example above, we would use a three-variable linear equation -- two independent variables and one dependent variable.

$$Y_C = A + B_1 X_1 + B_2 X_2$$

Where:

Y_c = The calculated or estimated value for the dependent (response) variable

A = The Y intercept, the value of Y when $X_1 = 0$ and $X_2 = 0$

X_1 = The first independent (explanatory) variable

B_1 = The slope of the line related to the change in X_1 , the value by which Y changes when X_1 changes by one.

X_2 = The second independent (explanatory) variable

B_2 = The slope of the line related to the change in X_2 , the value by which Y changes when X_2 changes by one.

Results of Audit Data Three-Variable Linear Regression AnalysisI. Using the above data on audit analysis and negotiated reductions, an analyst identified the following three variables:

X_1 = Computer Hours

X_2 = Field Audit Hours

Y = Cost Reductions Sustained

The results of analysts analysis are shown in the following table:

Regression Results		
Predictor Variable	Equation	r^2
Computer Hours	$Y = A + B X_1$.82
Field Audit Hours	$Y = A + B X_2$.60
Comp Hrs and Field Audit Hrs	$Y = A + B_1 X_1 + B_2 X_2$.88

You can see from the r^2 values in the above table that computer hours explains more of the variation in cost reduction recommendations sustained than is explained by field audit hours. If you had to select one independent variable, you would likely select Computer Hours. However, the combination of the two independent variables in multiple regression explains more of the variation in cost reduction recommendations sustained than the use of computer hours alone. The combination produces a stronger estimating tool.

Curvilinear Regression Analysis . In some cases, the relationship between the independent variable(s) may not be linear. Instead, a graph of the relationship on ordinary graph paper would depict a curve. You cannot directly fit a line to a curve using regression analysis. However, if you can identify a quantitative function that transforms a graph of the data to a linear relationship, you can then use regression analysis to calculate a line of best fit for the transformed data.

Common Transformation Functions	Examples
Reciprocal	$\frac{1}{X}$
Square Root	\sqrt{X}
Log-Log	$\log X$
Power	X^2

For example, improvement curve analysis (presented later in this text) uses a special form of curvilinear regression. While it can be used in price analysis and material cost analysis, the primary use of the improvement curve is to estimate labor hours. The curve assumes that less cost is required to produce each unit as the total units produced increases. In other words, the firm becomes more efficient as the total units produced increases.

There are many improvement curve formulations but one of the most frequently used is:

$$Y = AX^B$$

Where:

Y = Unit cost (in hours or dollars of the Xth unit)

X = Unit number

A = Theoretical cost of the first unit

B = Constant value related to the rate of efficiency improvement

Obviously, this equation does not describe a straight line. However, using the logarithmic values of X and Y (log-log transformation), we can transform this curvilinear relationship into a linear relationship for regression analysis. The result will be an equation in the form:

$$\log Y = \log A + B \log X$$

Where:

$\log Y$ = The logarithmic value of Y

$\log A$ = The logarithmic value of A

$\log X$ = The logarithmic value of X

We can then use the linear equation to estimate the logarithmic value of Y, and from that Y.

5.7 - Identifying Issues And Concerns

Questions to Consider in Analysis I. As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use regression analysis.

- ***Does the r^2 value indicate a strong relationship between the independent variable and the dependent variable?***

The value of r^2 indicates the percentage of variation in the dependent variable that is explained by the independent variable. Obviously, you would prefer an r^2 of .96 over an r^2 of .10, but there is no magic cutoff for r^2 that indicates that an equation is or is not acceptable for estimating purposes. However, as the r^2 becomes smaller, you should consider your reliance on any prediction accordingly.

- ***Does the T-test for significance indicate that the relationship is statistically significant?***

Remember that with a small data set, you can get a relatively high r^2 when there is no statistical significance in the relationship. The T-test provides a baseline to determine the significance of the relationship.

- ***Have you considered the prediction interval as well as the point estimate?***

Many estimators believe that the point estimate produced by the regression equation is the only estimate with which they need to be concerned. The point estimate is only the most likely estimate. It is part of a range of reasonable estimates represented by the prediction interval. The prediction interval is particularly useful in examining risk related to the estimate. A wide interval represents more risk than a narrow interval. This can be quite valuable in making decisions such as contract type selection. The prediction interval can also be useful in establishing positions for negotiation. The point estimate could be your objective, the lower limit of the interval your minimum position, and the upper limit your maximum position.

- ***Are you within the relevant range of data?***

The size of the prediction interval increases as the distance \bar{X} from increases. You should put the greatest reliance on forecasts made within the relevant range of existing data. For example, 12 is

within the relevant range when you know the value of Y for several values of X around 12 (e.g., 10, 11, 14, and 19).

- ***Are time series forecasts reasonable given other available information?***

Time series forecasts are all outside the relevant range of known data. The further you estimate into the future, the greater the risk. It is easy to extend a line several years into the future, but remember that conditions change. For example, the low inflation rates of the 1960s did not predict the hyper-inflation of the 1970s. Similarly, inflation rates of the 1970s did not predict inflation rates of the 1980s and 90s.

- ***Is there a run of points in the data?***

A run consisting of a long series of points which are all above or all below the regression line may occur when historical data are arranged chronologically or in order of increasing values of the independent variable. The existence of such runs may be a symptom of one or more of the following problems:

- - Some factor not considered in the regression analysis is influencing the regression equation (consider multivariate regression);
 - The equation being used in the analysis does not truly represent the underlying relationship between the variables;
 - The data do not satisfy the assumption of independence; or
 - The true relationship may be curvilinear instead of linear (consider curvilinear regression).
- ***Have you graphed the data to identify possible outliers or trends that cannot be detected through the mathematics of fitting a straight line?***

When you use 2-variable linear regression, you will fit a straight line through the data. However, the value of the relationship identified may be affected by one or more outliers that should not really be considered in your analysis. These can be easily identified through the use of a graph. Remember though, you cannot discard a data point simply because it does not fit on the line. The graph will help you identify an outlier, but you cannot discard it unless there is a valid reason (e.g., different methods were used for that item).

A graph can also permit you to identify situations where a single simple regression equation is not the best predictor. The graph may reveal that there is more than one trend affecting the data (e.g., the first several data points could indicate an upward trend, the latter data points a downward trend). It could also reveal the true relationship is a curve and not a straight line.

- ***Have you analyzed the differences between the actual and predicted values?***

Like the graph, this analysis will provide you information useful in identifying outliers (e.g., there may be one very large variance affecting the relationship). However, the outlier may not be as easy to identify as with a graph because the line will be pulled toward the outlier.

- *Are you comparing apples with apples?*

Regression analysis, like any technique based on historical data, assumes that the past is a good predictor of the future. For example, you might establish a strong relationship between production labor hours and quality assurance labor hours. However, if either production methods or quality assurance methods change substantially (e.g., automation) the relationship may no longer be of any value.

- *How current are the data used to develop the estimating equation?*

The more recent the data, the more valuable the analysis. Many things may have changed since the out-of-date data were collected.

- *Would another independent variable provide a better estimating tool?*

Another equation may produce a better estimating tool. As stated above, you would likely prefer an equation with an r^2 of .96 over one with an r^2 of .10.

- *Does the cost merit a more detailed cost analysis?*

If the cost is high and the r^2 is low, it may merit a more detailed analysis. For example, if you had a relatively low r^2 for a production labor effort, it may be worth considering the use of work measurement techniques in your analysis.

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6.0 - Chapter Introduction

In this chapter, you will learn to use moving averages to estimate and analyze estimates of contract cost and price.

Single Moving Average . If you cannot identify or you cannot measure an independent variable that you can use to estimate a particular dependent variable, your best estimate is often an average (mean) of past observations. The single moving average builds on this principle by defining the number of observations that you will consider. It assumes that the recent past is the best predictor of the future.

In a single moving average, data collected over two or more time periods (normally at least three) are summed and divided by the number of time periods. That average then becomes a

forecast for future time periods. As data from a new time period is added, data from an earlier time period is dropped from the average calculation. For example, a 12-month moving average uses data from the most recent 12 months. A 6-month moving average uses data from the most recent six months. You must determine the appropriate number of time periods to consider in the analysis. You can use any time period, but monthly data is the most common.

Double Moving Average. If you believe that there is a trend in the data, you can use a double moving average. A trend in the data means that the observation values tend to either increase or decrease over time. A double moving average requires that you calculate a moving average and then calculate a second moving average using the averages from your first moving average as observations.

6-Step Procedure for Using Moving Averages . When using moving averages, you should use the following 6-step procedure in your analysis:

Step 1. Collect the time services data.

Step 2. Determine which moving average model to use.

- No time-series trend -- use a single moving average.
- Time-series trend -- use a double moving average.

Step 3. Develop 1-period forecasts using different averaging periods to compare with actual observations to evaluate accuracy.

Step 4. Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Step 5. Select the averaging period found to produce the most accurate results.

Step 6. Use the moving average in forecasting.

6.1 - Identifying Situations For Use

Situations for Use . You can use moving averages in any situation where you are attempting to forecast a variable and you cannot identify or you cannot measure an independent variable except time that appears to be related to changes in the variable. In contract pricing, moving averages are often used to:

- **Develop estimating rates and factors.** For example, most production operations involve substantial amounts of material. When material is used, there is normally some amount of scrap that can no longer be used for its intended purpose. This can include:
 - Waste from production operations (e.g., sheet metal left over after shapes have been cut from it).

- Spoilage (e.g., material that has exceeded its useful shelf life, losses in storage, defective parts, etc.).
- Defective parts (e.g., parts that fail inspection during the production process).
- Material scrap rates are affected by a variety of factors including production methods, product design, and materials. Because the specific effect of these variables is difficult to identify and measure, scrap rates are commonly estimated using moving averages.
- Rates may be calculated in either dollars or units of material and are commonly calculated in one of the following ways:

$$\frac{\text{Scrap Dollars}}{\text{Total Assembly Material Dollars}} \text{ or } \frac{\text{Scrap Units}}{\text{Total Assembly Material Units}}$$

$$\frac{\text{Scrap Dollars}}{\text{Material Dollars Purchased}} \text{ or } \frac{\text{Scrap Units}}{\text{Material Unit Purchased}}$$

- In calculating such estimating factors, you should track the cost element being estimated (scrap) and the factor base separately over the averaging period. Then you can calculate the moving average rate by completing the rate calculation. However, if that is not possible, you can calculate the moving average using the scrap rates from each data period. The major disadvantage of the latter method is that periods of high and low production receive the same weight in analysis.
- **Estimate contractor sales volume.** A variety of factors affect sales volume (e.g., company products, the economy, Government spending, and many others). Estimates should include current contracts, known future contracts, other known sales, and currently unknown sales. One way of estimating currently unknown sales is with a moving average based on recent sales experience.
- **Estimate contract requirements .** Many contracts obligate the contractor to meet uncertain requirements. A requirements contract for a particular product may require the contractor to meet all Government demand during the contract period. A maintenance contract may require the contractor to respond to unscheduled service calls. In these and similar situations, a moving average can provide estimates of future requirements based on the recent past.
- **Estimate economic change.** A moving average can be used to estimate future economic change based on recent history. For example, wage rates and product price changes (index numbers) can be estimated using moving averages.

6.2 - Determining Which Moving Average Model To Use

General Criteria for Model Selection . There are several moving average models that can be used in contract pricing. The two most commonly used are the single moving average and the double moving average. Your decision on which model to use will depend on whether the data indicate a trend (upward or downward) in the values of the dependent variable. If there is:

- No time-series data trend - use a single moving average.
- Time-series data trend - use a double moving average.

Methods to Determine If Data Indicate Dependent Variable Trend . There are three common methods you could use to determine whether or not there is trend in a data set: graphic analysis, regression analysis, and Spearman's rank correlation coefficient.

- **Graphic Analysis.** Graphic analysis entails plotting the data (either manually or by computer) and determining by visual inspection whether or not there is trend in the data. The problem with this technique is that it is not consistently accurate. It is particularly difficult to make a decision when there may or may not be a slight trend.
- **Regression Analysis.** Regression analysis entails the calculation of a least-squares-best-fit (LSBF) estimating equation using time as the independent variable and testing the significance of the slope using the T-test. Though this is an accurate technique, it is rather tedious even when done using a computer.
- **Spearman's Rank Correlation Coefficient** . Spearman's rank correlation coefficient, also known as the Rank Spearman (RANSP) test involves calculation of an RS value and comparing that value with a critical value obtained from a table. This is the test that is most commonly used, because it is accurate and relatively easy to calculate even when done manually. However, before using the RANSP test, assure that the following three conditions have been met:
 - You must have data from at least four observations.
 - You must not have reason to suspect a cyclical or seasonal effect.
 - You must not have reason to suspect that there is a change in trend direction (monotonic trend).

Calculating the RANSP . A model to compute the RANSP is provided at RANSP Test Example 1

Assume that you have collected historical quarterly wage data and you want to determine if there is trend in the data.

Assume you have the following data in table format in the order in which the values occurred and number the time periods.

Quarter (t)	Wage Rate (Y)
1	\$12.50
2	\$11.80
3	\$12.85
4	\$13.95
5	\$13.30

6	\$13.95
7	\$15.00
8	\$16.20
9	\$16.10

Using the model at $***$, we compute an RS of .9375, which is greater than RS_{crit} of .4667. Thus, we can assume there is a trend in the data. Therefore, you should use a double moving average.

RANSP Test Example 2 . Again, assume that you have collected historical quarterly wage data and you want to determine if there is trend in the data.

Assume you have the following data in table format in the order in which the values occurred and number the time periods.

Quarter (t)	Wage Rate (Y)
1	\$12.70
2	\$12.60
3	\$12.00
4	\$13.00
5	\$12.10
6	\$12.50
7	\$12.80
8	\$13.00
9	\$12.85

In this example, the model computes an RS of .4375, which is less than RS_{crit} of .4667. Thus, we would assume that there is no trend in the data. Therefore, you would use a single moving average.

6.3 - Evaluating And Using Single Moving Averages

Procedures for Selecting a Single Moving Average for Forecasting . The single moving average is designed to smooth random variation in the estimate. The more data periods you use to calculate a single moving average, the greater the smoothing affect. For example, a 12-period moving average will average out most random variation, because each observation is only one-

twelfth of the average. However, a 12-period moving average will be slow to react to a true change in the variable that you are attempting to estimate. On the other hand, a three period moving average will react much faster, because one data point is one-third of the calculation instead of one-twelfth.

No averaging period is best for forecasting in all circumstances. You must identify the best averaging period for each situation:

Step 1. Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

- Use at least three periods of data in developing a moving average. You can calculate 3-period moving averages beginning in Period 3. You can calculate 4-period moving averages beginning in Period 4. For any value of n, you can calculate an n-period single moving average beginning in Period n.
- To conduct a meaningful evaluation of forecast accuracy, you must have at least two forecasts and actual data from the same periods for accuracy evaluation. As a result, the largest number of periods (n) that you can use for developing single moving averages is two less than the total number of observations.

Step 2. Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Step 3. Select the averaging period found to produce the most accurate results.

Calculations Required for Forecast Development . Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

Step 1A. Calculate Single Moving Averages .

Calculate single moving averages for available averaging periods using the following equation:

$$M1_{nt} = \frac{Y_{nt} + Y_{t-1} + \dots + Y_{t-n+1}}{n}$$

Where:

M_{nt} = A single n-period moving average calculated in Period t

Y_t = An observation in Period t of the variable being forecast

n = The number of time periods in the moving average

Step 1B. Develop Forecasts Using Moving Averages. Once you calculate a moving average, you can use that average for forecasting.

$$FM1_{n,t,t+h} = M1_{n,t}$$

Where:

$FM1_{n,t,t+h}$ = A single, n-period, moving average forecast made in Period t for Period t+h

n = The number of periods in the moving average

t = The period in which the forecast is made

h = The horizon, the number of periods you are forecasting into the future

Developing 1-Period Forecasts for Example 2 Data.

Step 1A. Calculate Single Moving Averages. In the previous section, we determined that we should use a single moving average to forecast future wage rates from the data below. Here we will use the data to demonstrate the procedures for single moving average forecast development.

Quarter (t)	Wage Rate (Y)
1	\$12.70
2	\$12.60
3	\$12.00
4	\$13.00
5	\$12.10
6	\$12.50
7	\$12.80
8	\$13.00
9	\$12.85

Note that we have observations from nine periods. That means that we can calculate 3-period, 4-period, 5-period, 6-period, and 7-period moving averages. With nine observations, we cannot evaluate forecasts based on a single moving average of more than seven (9 - 2) periods.

3-Period Single Moving Average			
Quarter (t)	Actual (Y)	³ Y	M1
1	\$12.70		
2	\$12.60		
3	\$12.00	\$37.30	\$12.43
4	\$13.00	\$37.60	\$12.53
5	\$12.10	\$37.10	\$12.37
6	\$12.50	\$37.60	\$12.53
7	\$12.80	\$37.40	\$12.47
8	\$13.00	\$38.30	\$12.77
9	\$12.85	\$38.65	\$12.88

* Complete terminology for these moving averages is M13,t. To save space the term has been simplified to M1.

Step 1B. Develop Forecasts Using Moving Averages : Once we calculate a single moving average, we can use that average to develop a forecast. To evaluate the accuracy of each moving average, we forecast one period into the future so that we can compare the forecast with the actual Y value. For example, the moving average from Period 3, becomes the Forecast for Period 4.

3-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	³ Y	M1	FM1**
1	\$12.70			
2	\$12.60			
3	\$12.00	\$37.30	\$12.43	
4	\$13.00	\$37.60	\$12.53	\$12.43
5	\$12.10	\$37.10	\$12.37	\$12.53
6	\$12.50	\$37.60	\$12.53	\$12.37
7	\$12.80	\$37.40	\$12.47	\$12.53
8	\$13.00	\$38.30	\$12.77	\$12.47
9	\$12.85	\$38.65	\$12.88	\$12.77

** Complete terminology for the forecasts in this column is FM13,t,t+1. To save space the term has been simplified to FM1.

We would develop the 4-period, 5-period, 6-period, and 7-period single moving average forecasts using the same procedure.

4-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	⁴ Y	M1	FM1
1	\$12.70			
2	\$12.60			
3	\$12.00			
4	\$13.00	\$50.30	\$12.58	
5	\$12.10	\$49.70	\$12.43	\$12.58
6	\$12.50	\$49.60	\$12.40	\$12.43
7	\$12.80	\$50.40	\$12.60	\$12.40
8	\$13.00	\$50.40	\$12.60	\$12.60
9	\$12.85	\$51.15	\$12.79	\$12.60
5-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	⁵ Y	M1	FM1
1	\$12.70			
2	\$12.60			
3	\$12.00			
4	\$13.00			
5	\$12.10	\$62.40	\$12.48	
6	\$12.50	\$62.20	\$12.44	\$12.48
7	\$12.80	\$62.40	\$12.48	\$12.44
8	\$13.00	\$63.40	\$12.68	\$12.48
9	\$12.85	\$63.25	\$12.65	\$12.68
6-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	⁶ Y	M1	FM1
1	\$12.70			

2	\$12.60			
3	\$12.00			
4	\$13.00			
5	\$12.10			
6	\$12.50	\$74.90	\$12.48	
7	\$12.80	\$75.00	\$12.50	\$12.48
8	\$13.00	\$75.40	\$12.57	\$12.50
9	\$12.85	\$76.25	\$12.71	\$12.57
7-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	⁷ Y	M1	F1
1	\$12.70			
2	\$12.60			
3	\$12.00			
4	\$13.00			
5	\$12.10			
6	\$12.50			
7	\$12.80	\$87.70	\$12.53	
8	\$13.00	\$88.00	\$12.57	\$12.53
9	\$12.85	\$88.25	\$12.61	\$12.57

Calculations Required for Evaluating Forecast Accuracy . Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

You can use several different statistics to measure the accuracy of a moving average forecast: the range of the error terms, the standard error of the forecast, or the mean absolute deviation of the forecast (MADf). Of these three options, the statistic which best combines the qualities of ease of computation and utility is the MADf. As a result, it is the statistic most commonly used to evaluate moving average accuracy.

You can use several different statistics to measure the accuracy of a moving average forecast: the range of the error terms, the standard error of the forecast, or the mean absolute deviation of the forecast (MADf). Of these three options, the statistic which best combines the qualities of ease of computation and utility is the MADf. As a result, it is the statistic most commonly used to evaluate moving average accuracy.

The MADf tells us on average how much, in absolute terms, actual values deviated from the forecasted value.

$$MAD_F = \frac{\sum |D|}{n}$$

Where:

MAD_F = The mean absolute deviation of the forecast

S = Summation of all the variables that follow the symbol

|D| = The absolute value of the deviation (i.e., the difference, without regard to sign) between the actual value which occurred and the value forecasted

$$|D| = |Y - F|$$

n = The number of deviations (Ds) computed

Evaluate forecast accuracy of single moving averages calculated using the data in Example 2.

3-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	 D
1	12.70				
2	12.60				
3	12.00	12.43			
4	13.00	12.53	12.43	0.57	0.57
5	12.10	12.37	12.53	-0.43	0.43
6	12.50	12.53	12.37	0.13	0.13
7	12.80	12.47	12.53	0.27	0.27
8	13.00	12.77	12.47	0.53	0.53
9	12.85	12.88	12.77	0.08	0.08
Total Absolute Deviation					2.01
Mean Absolute Deviation = $\frac{ D }{n} = \frac{2.01}{6} =$					0.34
4-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	 D

1	12.70				
2	12.60				
3	12.00				
4	13.00	12.58			
5	12.10	12.43	12.58	-0.48	0.48
6	12.50	12.40	12.43	0.07	0.07
7	12.80	12.60	12.40	0.40	0.40
8	13.00	12.60	12.60	0.40	0.40
9	12.85	12.79	12.60	0.25	0.25
Total Absolute Deviation					1.60
Mean Absolute Deviation = $ D n = 1.60 / 5 =$					0.32
5-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	 D
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10	12.48			
6	12.50	12.44	12.48	0.02	0.02
7	12.80	12.48	12.44	0.36	0.36
8	13.00	12.68	12.48	0.52	0.52
9	12.85	12.65	12.68	0.17	0.17
Total Absolute Deviation					1.07
Mean Absolute Deviation = $ D n = 1.07 / 4 =$					0.27
6-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	 D
1	12.70				
2	12.60				
3	12.00				

4	13.00				
5	12.10				
6	12.50	12.48			
7	12.80	12.50	12.48	0.32	0.32
8	13.00	12.57	12.50	0.50	0.50
9	12.85	12.71	12.57	0.28	0.28
Total Absolute Deviation					1.10
Mean Absolute Deviation = $ D n = 1.10 / 3 =$					0.37
7-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	 D
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10				
6	12.50				
7	12.80	12.53			
8	13.00	12.57	12.53	0.47	0.47
9	12.85	12.61	12.57	0.28	0.28
Total Absolute Deviation					0.75
Mean Absolute Deviation = $ D n = .75 / 2 =$					0.32

Selecting an Averaging Period . Select the averaging period found to produce the most accurate results.

Summary Of MAD Computations	
n	MAD_F
3	0.34
4	0.32
5	0.27

6	0.37
7	0.38

The lowest MAD_F in this example was attained using a 5-period single moving average. Accordingly, you should select a 5-period single moving average for forecasting.

Use the Single Moving Average in Forecasting . Use the moving average with the lowest MAD_F for forecasting. Based on an evaluation of the data in Example 2, you should use the most recent 5-period single moving average to forecast for any future period. For example, the forecast for Period 13 would be \$12.65.

The selection of the most accurate averaging period for forecast development is essential. Different averaging periods can produce substantially different forecasts. For example, using different averaging periods and the data in this example, you could have calculated a wide range of forecasts for Period 13.

Period 13 Forecast Comparison	
n	FM1_{n,9,13}
3	\$12.88
4	\$12.79
5	\$12.65
6	\$12.71
7	\$12.61

Of these possibilities, the 5-period single moving average forecast, \$12.65, appears to be the most reasonable.

6.4 - Evaluating And Using Double Moving Averages

Procedures for Selecting a Double Moving Average for Forecasting. The double moving average is designed to develop a forecast that smoothes random variation and projects any trend exhibited in the data. As with the single moving average, no averaging period is best for forecasting in all circumstances. You must identify the best averaging period for each situation:

Step 1. Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

- Normally, we use at least a three period double moving average. Since a double moving average is a moving average of moving averages, you cannot begin to calculate a 3-

period double moving average until Period 5. You can calculate 4-period double moving averages beginning in Period 7. For any value of n, you can calculate an n-period double moving average beginning in Period 2n - 1.

- To conduct a meaningful evaluation of forecast accuracy, you must have at least two forecasts and actual data for the same period for accuracy evaluation. As a result, you must have 2n + 1 data points in order to calculate a double moving average forecast and the related MADF.

Step 2. Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Step 3. Select the averaging period found to produce the most accurate results.

Calculations Required for Forecast Development . Develop 1-period forecasts using available averaging periods so that you can compare forecasts with actual observations to evaluate accuracy.

Step 1A. Calculate Double Moving Averages . Calculate double moving averages for available averaging periods using the following equation:

$$M2_{n,t} = \frac{M1_{n,t} + M1_{n,t-1} + \dots + M1_{n,t-n+1}}{n}$$

Where:

$M2_{n,t}$ = An n-period double moving average calculated in Period t

$M1_{n,t}$ = An n-period single moving average calculated in period t

n = Number of periods in the moving average

Note : You must use the same value of n for calculating both M1 and M2.

Step 1B. Develop Forecasts Using Moving Averages : Once you calculate a double moving average, you can use that average to develop a forecast.

$$FM2_{n,t,t+h} = A_{n,t} + B_{n,t} + h$$

Where:

$FM2_{n,t,t+h}$ = The, n-period, double moving average forecast made in period t for period t+h

$A_{n,t}$ = The intercept for an n-period double moving average forecast, calculated:

$$A_{n,t} = 2M1_{n,t} - M2_{n,t}$$

$B_{n,t}$ = The slope for an n-period double moving average forecast, calculated:

$$B_{n,t} = \frac{2}{n-1} (M1_{n,t} - M2_{n,t})$$

n = The number of periods in the moving average

t = The period in which the forecast is made

h = The horizon, the number of periods you are forecasting into the future

Note : Depending on the value of n and the period in which the forecast is made, there is a unique intercept ($A_{n,t}$) and slope ($B_{n,t}$).

Developing 1-Period Forecasts for Example 1 Data .

Step 1A. Calculate Double Moving Averages . In the previous section, we determined that we should use a double moving average to develop a forecast from the data below. Here we will use the data to demonstrate the procedures for double moving average forecast development.

Quarter (t)	Wage Rate (Y)
1	\$12.50
2	\$11.80
3	\$12.85
4	\$13.95
5	\$13.30
6	\$13.95
7	\$15.00
8	\$16.20
9	\$16.10

3Y	M1	3M1	M2
37.15	12.38		
38.60	12.87		
40.10	13.37	38.62	12.87

41.20	13.73	39.97	13.32
42.25	14.08	41.18	13.73
45.15	15.05	42.86	14.29
47.30	15.77	44.90	14.97

Step 1B. Develop Forecasts Using Moving Averages . Period 5 is the first period that we can develop a 3-period double moving average forecast, because that is the first period that we have the values of M1 and M2 that we need to make the forecast. In Period 5, we can make a forecast for Period 6 as follows:

$$\begin{aligned}
 A_{3,5} &= 2M1_{3,5} - M2_{3,5} \\
 &= 2(13.37) - 12.87 \\
 &= 26.74 - 12.87 \\
 &= 13.87
 \end{aligned}$$

$$\begin{aligned}
 B_{3,5} &= (2(3-1))(M1_{3,5} - M2_{3,5}) \\
 &= (2(3-1))(13.37 - 12.87) \\
 &= (2(2))(13.37 - 12.87) \\
 &= 13.37 - 12.87 \\
 &= .50
 \end{aligned}$$

$$\begin{aligned}
 FM2_{3,5,6} &= A_{3,5} + B_{3,5} (h) \\
 &= 13.87 + .50(1) \\
 &= 13.87 + .50 \\
 &= 14.37
 \end{aligned}$$

3-Period Double Moving Average Forecast						
t	Actual	M1	M2	A	B	FM2
1	12.50					
2	11.80					
3	12.85	12.38				
4	13.95	12.87				
5	13.30	13.37	12.87	13.87	0.50	
6	13.95	13.73	13.32	14.14	0.41	14.37
7	15.00	14.08	13.73	14.43	0.35	14.55
8	16.20	15.05	14.29	15.81	0.76	14.78
9	16.10	15.77	14.97	16.57	0.80	16.57

Forecasts developed using a 4-period moving average and the same procedures are shown in the following table. Note that only two forecasts can be made for comparison with actual observations.

4-Period Double Moving Average Forecast						
t	Actual	M1	M2	A	B	FM2
1	12.50					
2	11.80					
3	12.85					
4	13.95	12.78				
5	13.30	12.98				
6	13.95	13.51				
7	15.00	14.05	13.33	14.77	0.48	
8	16.20	14.61	13.79	15.43	0.55	15.25
9	16.10	15.31	14.37	16.25	0.63	15.98

Calculations Required for Evaluating Forecast Accuracy . Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations. Here we use the same formula for calculating MAD_F that we used in evaluating the accuracy of single moving averages.

$$MAD_F = \frac{\sum |D|}{n}$$

Where:

MAD_F = The mean absolute deviation of the forecast

S = Summation of all the variables that follow the symbol

|D| = The absolute value of the deviation (i.e., the difference, without regard to sign) between the actual value which occurred and the value forecasted

$$|D| = |Y - F|$$

n = The number of deviations (Ds) computed

3-Period Double Moving Average Forecast Evaluation

t	Actual	M1	M2	A	B	FM2	D	 D
1	12.50							
2	11.80							
3	12.85	12.38						
4	13.95	12.37						
5	13.30	13.37	12.87	13.87	0.50			
6	13.95	13.73	13.32	14.14	0.41	14.37	-0.42	0.42
7	15.00	14.08	13.73	14.43	0.35	14.55	0.45	0.45
8	16.20	15.05	14.29	15.81	0.76	14.78	1.42	1.42
9	16.10	15.77	14.97	16.57	0.80	16.57	-0.47	0.47
Total Absolute Deviation								2.76
Mean Absolute Deviation = $ D n = 2.76 / 4 =$								0.69
4-Period Double Moving Average Forecast Evaluation								
	Actual	M1	M2	t	B	FM2	D	 D
1	12.50							
2	11.80							
3	12.85							
4	13.95	12.78						
5	13.30	12.98						
6	13.95	13.51						
7	15.00	14.05	13.33	14.77	0.48			
8	16.20	14.61	13.79	15.43	0.55	15.25	0.95	0.95
9	16.10	15.31	14.37	16.25	0.63	15.98	0.12	0.12
Total Absolute Deviation								1.07
Mean Absolute Deviation = $ D n = 1.07 / 2 =$								0.69

Select Averaging Period

Select the averaging period found to produce the most accurate results.

Summary Of MAD Computations

n	MAD_F
3	0.69
4	0.54

The lowest MAD_F in this example was attained using a 4-period double moving average. Accordingly, you should select a 4-period double moving average for forecasting.

Use a Double Moving Average in Forecasting . Use the moving average with the lowest MAD_F for forecasting. Based on our evaluation of the data in Example 1, we would use the 4-period double moving average for forecasting. For example our forecast for Period 13 [four periods (h) into the future] would be \$18.77, calculated as follows:

$$\begin{aligned}
 A_{3,5} &= 2M1_{4,9} - M2_{4,9} \\
 &= 2(15.31) - 14.37 \\
 &= 30.62 - 14.37 \\
 &= 16.25
 \end{aligned}$$

$$\begin{aligned}
 B_{3,5} &= (2 (4-1)) (M1_{4,9} - M2_{4,9}) \\
 &= (2 (4-1)) (15.31 - 14.37) \\
 &= (2 3) (15.31 - 14.37) \\
 &= (2 3) (.94) \\
 &= .63
 \end{aligned}$$

$$\begin{aligned}
 FM2_{3,5,6} &= A_{4,9} + B_{4,9} (h) \\
 &= 16.25 + .63 (4) \\
 &= 16.25 + 2.52 \\
 &= 18.77
 \end{aligned}$$

The selection of the most accurate averaging period for forecast development is essential. For example, using different averaging periods and the data in this example, we could have calculated two very different forecasts.

Forecast Comparison	
n	FM2_{n,9,13}
3	\$19.77
4	\$18.77

6.5 - Identifying Issues And Concerns

Questions to Consider in Analysis . As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use moving averages.

- ***Is a moving average the best choice for estimate development?***

When using a moving average, you assume that the trend experienced over time is the best guide available to forecast future variable values. If that assumption is not correct, you should use another technique. Detailed estimates that consider all the facts involved are normally more defensible in negotiations than the result of any estimating relationship. If an independent variable (other than time) can be identified and measured, another comparison technique may provide better results than moving average analysis. For example, estimating parts demand based on sales and usage data would probably produce better results than an estimate based on use of a moving average. A moving average can estimate price changes based on recent periods but it cannot predict a turning point that will alter the historical pattern.

- ***Is the type of moving average selected appropriate for the situation?***

When there is a trend in the data, you should use a double moving average. When there is no trend, you should use a single moving average. If you use a single moving average in a situation where a trend exists, your forecast will not consider the trend.

- ***Is the averaging period the best choice for the data?***

You should select the averaging period that provides the best estimates when tested against actual observations. Take special care in your analysis when the moving average covers a large number or periods (e.g., 12 months). Selection of an average that covers a large number of periods is often appropriate because it dampens the effect of random fluctuation. However, an average that considers a large number of data points will also make it more difficult to identify a trend in the data. Occasionally, an estimator will use a large number of periods to mask a trend in the data. When analyzing estimates made using a moving average, you should look at the raw data and consider appropriate alternative estimating procedures.

- ***How far into the future can you forecast?***

The farther into the future that you forecast, the greater the risk. Remember, that you cannot predict a change in an historical trend with a moving average.

In this chapter, you will learn improvement curve concepts and their application to cost and price analysis.

- 7.0 - [Chapter Introduction](#)
- 7.1 - [Situations for Use](#)
- 7.2 - [Analyzing Improvement Using Unit Data and Unit Theory](#)
- 7.3 - [Analyzing Improvement Using Lot Data and Unit Theory](#)
- 7.4 - [Fitting a Unit Curve](#)
- 7.5 - [Estimating Using Unit Improvement Curve Tables](#)
- 7.6 - [Identifying Issues and Concerns](#)

7.0 Chapter Introduction

7.0.1 Basic Improvement Curve Concept

You may have learned about improvement curves using the name learning curve analysis. Today, many experts feel that the term learning curve implies too much emphasis on learning by first-line workers. They point out that the theory is based on improvement by the entire organization not just first-line workers. Alternative names proposed for the theory include: improvement curve, cost-quantity curve, experience curve, and others. None have been universally accepted. In this text, we will use the term improvement curve to emphasize the need for efforts by the entire organization to make improvements to reduce costs.

Just as there are many names for the improvement curve, there are many different formulations. However, in each case the general concept is that the resources (labor and/or material) required to produce each additional unit declines as the total number of units produced over the entire production history increases. The concept further holds that decline in unit cost can be predicted mathematically. As a result, improvement curves can be used to estimate contract price, direct labor-hours, direct material cost, or any other recurring contract cost.

7.0.2 Improvement Curve History

The improvement curve is based on the concept that, as a task is performed repetitively, the time required to perform the task will decrease. Management planners have followed that element of the concept for centuries, but T. P. Wright pioneered the idea that improvement could be estimated mathematically. In February 1936, Wright published his theory in the *Journal of Aeronautical Sciences* as part of an article entitled "Factors Affecting the Cost of Airplanes." Wright's findings showed that, as the number of aircraft produced in sequence increased, the direct labor input per airplane decreased in a regular pattern that could be estimated mathematically.

During the mobilization for World War II, both aircraft companies and the Government became interested in the theory. Among other considerations, the theory implied that a fixed amount of labor and equipment could be expected to produce larger and larger quantities of defense products as production continued.

After World War II, the Government engaged the Stanford Research Institute (SRI) to study the validity of the improvement curve concept. The study analyzed essentially all World War II airframe direct labor input data to determine whether there was sufficient evidence to establish a standard estimating model. The SRI study validated a mathematical model based on the World

War II findings that could be used as a tool for price analysis. However, that model was slightly different than the one originally offered by Wright.

Since World War II, the improvement curve concept has been used by Government and industry to aid in pricing contracts. Over the years, the improvement curve has been used as a contract estimating and analysis tool in a variety of industries including: airframes, electronics systems, machine tools, shipbuilding, missile systems, and depot level maintenance of equipment. Improvement curves have also been applied to service and construction contracts where tasks are performed repetitively.

7.0.3 Identifying Basic Improvement Curve Theories: Unit Improvement Curve

Since 1936, many different formulations have been proposed to explain and estimate the improvement that takes place in repetitive production efforts. Of these, the two most popular are the unit improvement curve and the cumulative average improvement curve

Unit Improvement Curve. The unit improvement curve is the model validated by the post-World War II SRI study. The formulation is also known by two other names: Crawford curve, after one of the leaders of the SRI research; and Boeing curve, after one of the firms that first embraced its use.

Unit curve theory can be stated as follows:

As the total volume of units produced doubles the cost per unit decreases by some constant percentage.

The constant percentage by which the costs of doubled quantities decrease is called the rate of learning. The term "slope" in the improvement curve analysis is the difference between 100 percent and the rate of improvement. If the rate of improvement is 20 percent, the improvement curve slope is 80 percent (100 percent - 20 percent). The calculation of slope is described in detail later in the chapter.

Unit curve theory is expressed in the following equation:

$$Y = AX^B$$

Where:

Y = Unit cost (hours or dollars) of the Xth unit

X = Unit number

A = Theoretical cost (hours or dollars) of the first unit

B = Constant that is related to the slope and the rate of change of the improvement curve. It is calculated from the relationship:

$$B = \frac{\text{Logarithm of the Slope}}{\text{Logarithm of 2}}$$

In calculating B, the slope MUST be expressed in decimal form rather than percentage form. Then B will be a negative #, leading to the decreasing property stated above.

7.0.4 Identifying Basic Improvement Curve Theories: Cumulative Average Improvement Curve

Cumulative Average Improvement Curve. The cumulative average improvement curve is the model first introduced by Wright in 1936. Like the unit improvement curve, the cumulative average curve is also known by two other names: Wright Curve, after T.P. Wright; and Northrop Curve, after one of the firms that first embraced its use.

Cumulative average theory can be stated as follows:

As the total volume of units produced doubles the average cost per unit decreases by some constant percentage.

As with the unit improvement curve, the constant percentage by which the costs of doubled quantities decrease is called the rate of improvement. The slope of the improvement curve analysis is the difference between 100 percent and the rate of learning. However, the rate of improvement and the slope are measured using cumulative averages rather than the unit values used in unit improvement curve analysis.

Cumulative average curve theory is expressed in the following equation:

$$Y = AX^B$$

Where:

\bar{Y} = Cumulative average unit cost (hours or dollars) of units

through the X^{th} unit

All other symbols have the same meaning used in describing the unit improvement curve.

Curve Differences. Note that the only difference between definitions of the unit improvement curve and the cumulative average improvement curve theories is the word **average**. In the unit curve, unit cost is reduced by the same constant percentage. In the cumulative average curve, the cumulative average cost is reduced by the same constant percentage.

The most significant practical difference between the two different formulations is found in the first few units of production. Over the first few units, an operation following the cumulative average curve will experience a much greater reduction in cost (hours or dollars) than an operation following a unit curve with the same slope. In later production, the reduction in cost for an operation following a cumulative average curve will be about the same as an operation following a unit curve with the same slope.

Because of the difference in early production, many feel that the unit curve should be used in situations where the firm is fully prepared for production; and the cumulative average curve should be used in situations where the firm is not completely ready for production. For example, the cumulative average curve should be used in situations where significant tooling or design problems may NOT be completely resolved. In such situations, the production of the first units will be particularly inefficient but improvement should be rapid as problems are resolved.

In practice, firms typically use one formulation regardless of differences in the production situation. Most firms in the airframe industry use the cumulative average curve. Most firms in other industries use the unit curve.

7.1 Identifying Situations for Use

The improvement curve cannot be used as an estimating tool in every situation. Situations that provide an opportunity for improvement or reduction in production hours are the types of situations that lend themselves to improvement curve application. Use of the improvement curve should be considered in situations where there is:

- **A high proportion of manual labor.**

It is more difficult to reduce the labor input when there is limited labor effort, the labor effort is machine paced, or individual line workers only touch the product for a few seconds.

- **Uninterrupted production.**

As more and more units are produced the firm becomes more adept at production and the labor hour requirements are reduced. If supervisors, workers, tooling, or other elements of production are lost during a break in production, some improvement will also likely be lost.

- **Production of complex items.**

The more complex the item the more opportunity there is to improve.

- **No major technological change.**

The theory is based on continuing minor changes in production and in the item itself. However, if there are major changes in technology, the benefit of previous improvement may be lost.

- **Continuous pressure to improve.**

The improvement curve does not just happen; it requires management effort. The management of the firm must exert continuous pressure to improve. This requires investment in the people and equipment needed to obtain improvement.

7.1.1 Situations for Use

As you examine situations that appear to have potential for improvement curve application, consider management emphasis on the following factors affecting the rate of improvement:

- **Job Familiarization By Workers.**

As noted earlier, many feel that this element has been overemphasized over the years. Still, workers do improve from repetition and that improvement is an important part of the improvement curve.

- **Improved Production Procedures.**

As production continues, both workers and production engineers must constantly be on the lookout for better production procedures.

- **Improved Tooling and Tool Coordination.**

Part of the examination of production procedures must consider the tooling used for production. Tooling improvements offer substantial possibilities for reduction of labor requirements.

- **Improved Work Flow Organization.**

Improving the flow of the work can substantially reduce the labor effort that does not add value to the product. Needless movement of work in progress can add significant amounts of labor effort.

- **Improved Product Producibility.**

Management and workers must constantly consider product changes that will make the product easier to produce without degrading the quality of the final product.

- **Improved Engineering Support.**

The faster production problems can be identified and solved, the less production labor effort will be lost waiting for problem resolution.

- **Improved Parts Support.**

As production continues, better scheduling should be possible to eliminate or significantly reduce worker time lost waiting for supplies. In addition, production materials more appropriate for production can be identified and introduced to the production process.

7.1.2 Factors that Support Improvement

As you examine situations that appear to have potential for improvement curve application, consider management emphasis on the following factors affecting the rate of improvement:

- **Job Familiarization By Workers** . As noted earlier, many feel that this element has been overemphasized over the years. Still, workers do improve from repetition and that improvement is an important part of the improvement curve.
- **Improved Production Procedures** . As production continues, both workers and production engineers must constantly be on the lookout for better production procedures.
- **Improved Tooling and Tool Coordination** . Part of the examination of production procedures must consider the tooling used for production. Tooling improvements offer substantial possibilities for reduction of labor requirements.
- **Improved Work Flow Organization** . Improving the flow of the work can substantially reduce the labor effort that does not add value to the product. Needless movement of work in progress can add significant amounts of labor effort.
- **Improved Product Producibility** . Management and workers must constantly consider product changes that will make the product easier to produce without degrading the quality of the final product.
- **Improved Engineering Support** . The faster production problems can be identified and solved, the less production labor effort will be lost waiting for problem resolution.
- **Improved Parts Support** . As production continues, better scheduling should be possible to eliminate or significantly reduce worker time lost waiting for supplies. In addition, production materials more appropriate for production can be identified and introduced to the production process.

7.2 Analyzing Improvement Using Unit Data and the Unit Theory

Section Introduction

In this text, we will only consider application of the unit improvement curve in making initial contract estimates. There are many texts that address other improvement curve theories (e.g., cumulative average improvement curves), as well as many advanced issues such as the effects of contract changes, breaks in production, and retained learning.

Improvement Illustration

To illustrate the effect of the unit curve, assume that the first unit required 100,000 labor-hours to produce. If the slope of the curve is 80 percent slope, the following table demonstrates the labor-hours required to produce units at successively doubled quantities.

Units Produced	LABOR-HOURS Per Unit at Doubled Quantities	Difference in LABOR-HOURS Per Unit at Doubled Quantities	Rate of Improvement (%)	Slope of Curve (%)
1	100,000			
2	80,000	20,000	20	80
4	64,000	16,000	20	80
8	51,200	12,800	20	80
16	40,960	10,240	20	80
32	32,768	8,192	20	80

Obviously, the amount of labor-hour reduction between doubled quantities is not constant. The number of hours of reduction between doubled quantities is constantly declining. However, the rate of change or decline remains constant (20 percent).

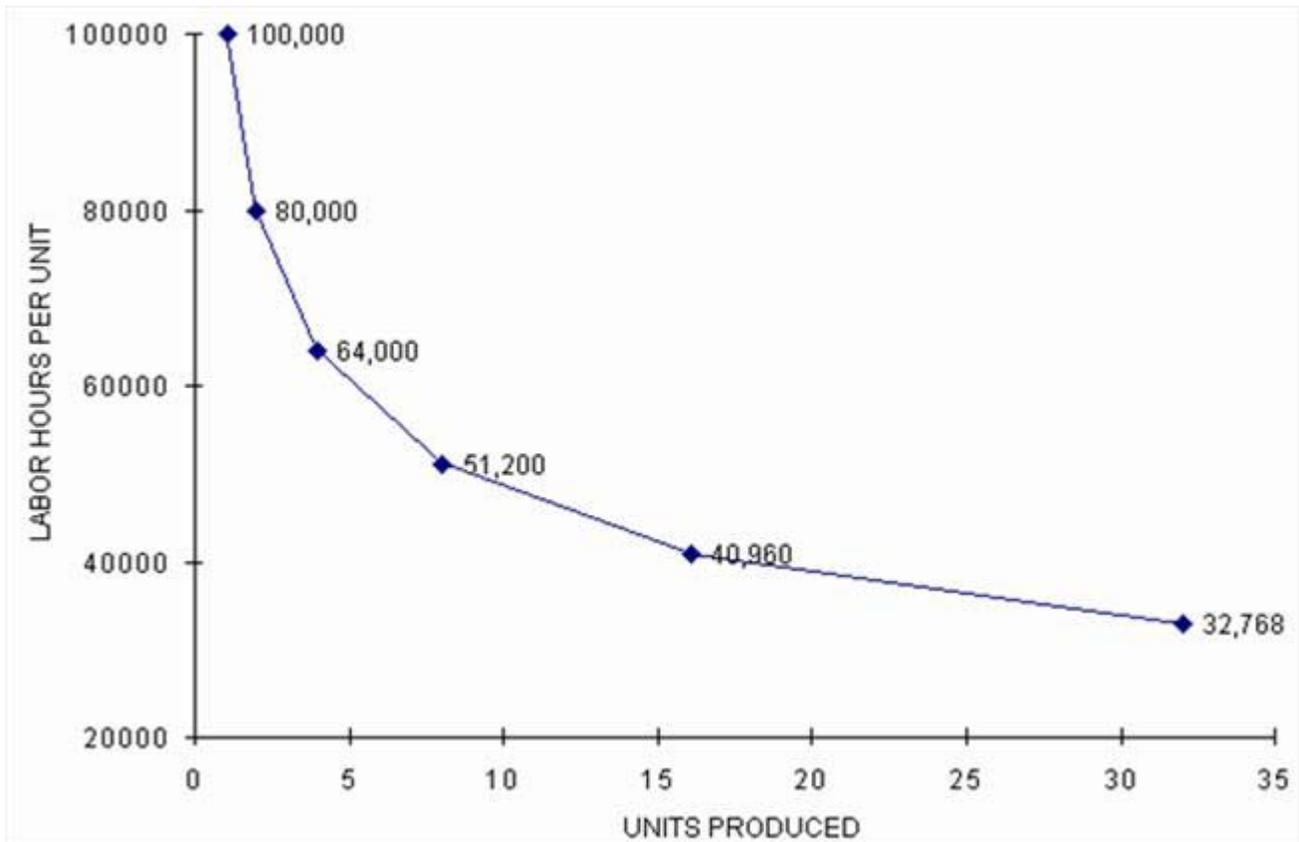
Also note that the number of units required to double the quantity produced is constantly increasing. Between Unit #1 and Unit #2, it takes only one unit to double the quantity produced. Between Unit #16 and Unit #32, 16 units are needed.

7.2.1 Analysis Techniques Graphing the Data

Improvement curves are modeled using an exponential equation. Exponential equations can be solved using logarithms. Plotting improvement curve data on a logarithmic scale causes the points to lie on a straight line, which allows for projection of future costs. Historically this process was accomplished manually; however, it can now be accomplished using the regression function on most data analysis tools or using computer modeling programs.

Since it is often helpful to understand the theory behind the use of data analysis tools and computer models, this text will include information and illustrations on how to manually calculate the cost of future units as well as introduce the reader to the differences when calculating using the regression function.

Rectangular Coordinate Graph. A labor-hour graph of this data curve drawn on ordinary graph paper (rectangular coordinates) becomes a curve as shown in the graph below. In this graph equal spaces represent equal amounts of change. When thinking of numbers in terms of their absolute values, this graph presents an accurate picture, but it is difficult to make an accurate prediction from this curve.



The graph is a curve because the number of hours of reduction between doubled quantities is constantly declining and an increasing number of units are required to double the quantity produced. Note that most of the improvement takes place during the early units of production. The curve will eventually become almost flat. The number of production hours could become quite small but it will never reach zero.

Log-Log Graph. To examine the data and make predictions using unit improvement curve theory, we need to transform the data to logarithms. One way of making the transformation is through the use of log-log graph paper also known as full-logarithmic graph paper.

7.2.2 Log-Log Paper

There are several special elements that we must consider when using log-log graph paper.

- There is already a scale indicated on both the horizontal and vertical axes. Note that there are no zeros. Values can approach zero but never reach it.
- The scale only goes from "1" to "1". Each time the number scale goes from "1" to "1", the paper depicts a cycle. Each "1" moving up on the vertical axis or to the right on the horizontal axis is 10 times the "1" before it. You should mark the actual scale you are using in the margin of the log-log paper before starting to plot points.
- In improvement curve analysis, always graph the number of the unit produced on the horizontal axis. Assign the first "1" on the left of the page a value of 1 representing the first unit produced. The second "1" is 10. The third "1" is 100. The fourth "1" is 1,000.
- Always graph the cost in hours or dollars on the vertical axis. The scale will change depending on the data being graphed. The first "1" can be .001, .01, 1, 100, 1,000 or any other integral power of 10. Whatever the value assigned to the first "1," the next "1" is 10 times more, and the next one 10 times more than that. To determine the scale to be used:
 - Estimate the largest number to be plotted or read on the Y axis. This figure will probably be the theoretical cost of the first unit. For example, suppose this is 60,000 hours.
 - Determine the next integral power of ten above this number (e.g., the next integral power above 60,000 is 100,000).
 - Assign this value to the horizontal line at the top of the upper cycle on the Y axis. The horizontal line at the top of the next lower cycle must then represent 10,000 of the same units, and the line at the bottom of the lower cycle represents 1,000.
- On log-log graph paper, the distances between numbers on each axis are equal for equal percentage changes. For example, the distance between "1" and "2" is the same as between "4" and "8;" both represent a 100 percent increase.

7.2.3 Log-Log Graph

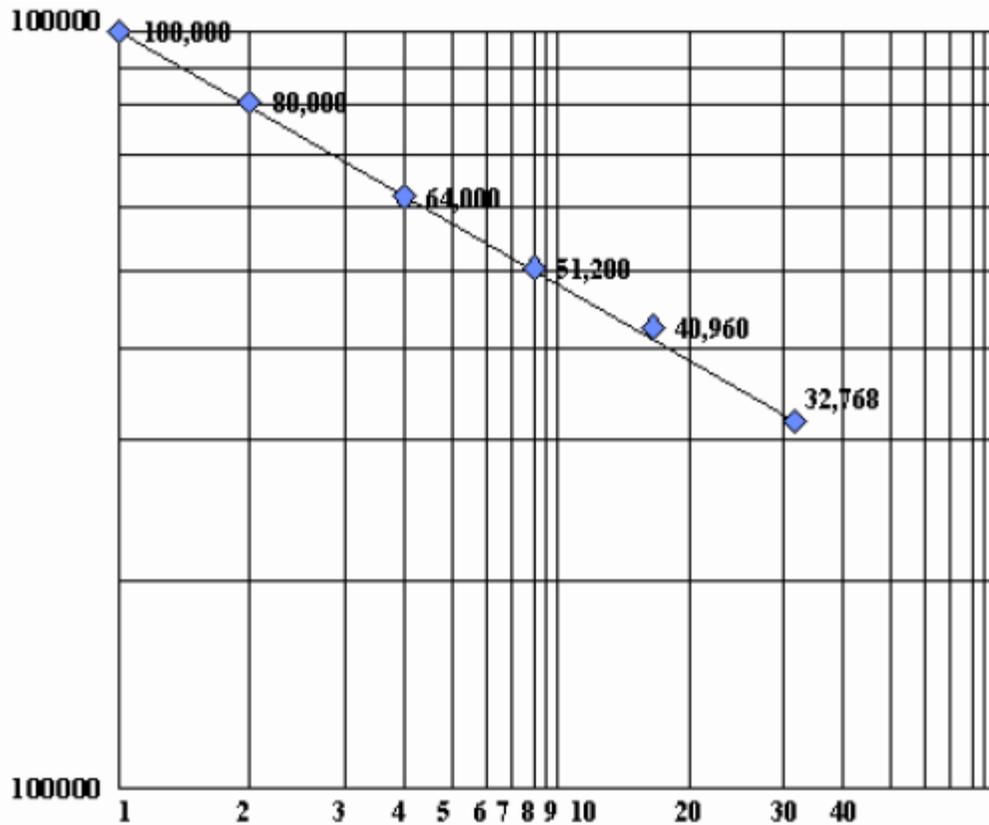
You can obtain surprisingly accurate results from a log-log graph, but your accuracy greatly depends on your graphing technique.

- Always use a sharp pencil.
- Make points plotted on the paper as small as possible and the lines as narrow as possible.
- When the smallest possible point has been marked on the paper, it may easily be lost sight of or confused with a blemish in the paper. To avoid this, draw a small ring around the point. Circles, triangles, and squares are also used to identify points which belong to different sets of data.

- Exercise great care in drawing a line. If it is supposed to go through a point, it should pass exactly through it, not merely close to it.

A graph of the data described in the example above forms a perfectly straight line when plotted on log-log paper. That is, a straight line passes exactly through each of the points. A straight line on log-log paper indicates that the rate of change is constant.

Since improvement curve theory assumes continuing improvement at a constant rate, the straight line becomes an excellent estimating tool. Assuming that improvement will continue at the same rate, the line can be extended to estimate the cost of future units.



7.2.4 Calculating the Theoretical Value of Unit #1

When we discuss improvement curves, we normally describe them in terms of the theoretical value for Unit #1 and the slope of the curve. With these two values, you can use graph paper, tables, or computer programs to estimate the cost of future units.

The value of Unit #1 is referred to as a **theoretical value** (T1), because in most cases you will not know the actual cost of Unit #1. Instead, T1 is the value indicated by the line-of-best-fit. On a graph, it is the point where the line-of-best-fit and the vertical line representing Unit #1 intersect. **(Remember, the graph of the improvement curve always begins with Unit #1.)**

7.2.5 Estimating the Slope

The term "slope" as used for improvement curves is a mathematical misnomer. It cannot be related to the definition of slope in a straight line on rectangular coordinates. Instead, the slope of an improvement curve is equal to 100 minus that constant percentage decrease (100 - rate of improvement).

You can calculate the slope of a curve, by dividing the unit cost (Y_X) at some unit (X) into the unit cost (Y_{2X}) at twice the quantity ($2X$) and multiplying the resulting ratio by 100.

Therefore, you can measure the slope of an improvement curve drawn on log-log paper by reading a cost (Y_X) at any quantity, X ; reading a cost (Y_{2X}) at any quantity, $2X$; dividing the second value by the first; and multiplying by 100.

For example, if the number of hours to make Unit #5 is 70 and the number of hours to make Unit #10 is 50, the slope of the improvement curve is:

$$\text{Slope} = 100 \left(\frac{Y_{2X}}{Y_X} \right)$$

$$\text{Slope} = 100 (Y_{10} / Y_5)$$

$$\text{Slope} = 100 (50 / 70)$$

$$\text{Slope} = 71.4 \text{ percent}$$

Slope Research Data. The post-war SRI study revealed that many different slopes were experienced by different firms, sometimes by different firms manufacturing the same products. In fact, manufacturing data collected from the World War II aircraft manufacturing industry had slopes ranging from 69.7 percent to almost 100 percent. These slopes averaged 80 percent.

Research by DCAA in 1970 found curves ranging from less than 75 percent to more than 95 percent. The average slope was 85 percent.

Slope Selection and Verification. Unfortunately, information on industry average curves is frequently misapplied by practitioners who use them as a standard or norm. Because each situation is different, you should select a slope based on your analysis of the situation and not on general averages. The order of preference in slope selection is:

- A curve developed from data pertaining to the production of the same product (as we did above).
- The median percentage from a group of curves for items having some similarity to the end item.
- The median percentage from the product category in which the item would most likely be included.

7.2.6 Estimating the Cost/Price of Future Units

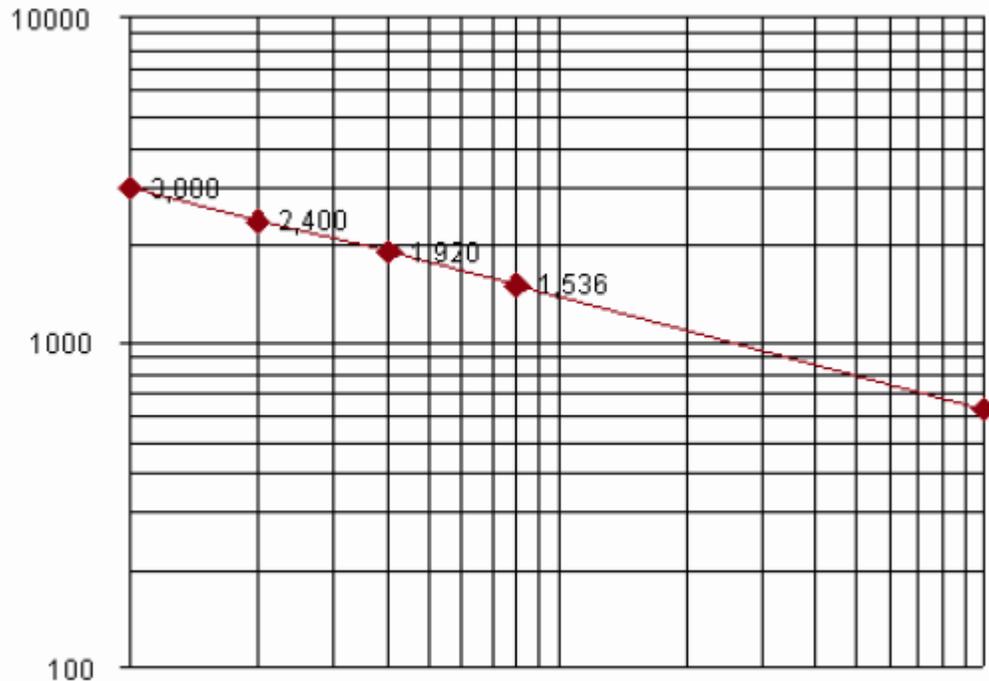
The primary purpose for estimating an improvement curve is to predict the cost of future production. The prediction is based on the assumption (not always true) that the past is a good predictor of the future. In terms of the unit improvement curve theory, this assumption means that the unit cost (hours or dollars) of doubled quantities will continue to decrease by the same constant percentage.

Using a graph, you can predict future costs by drawing a line-of-best-fit through the historical data graphed on log-log paper and extending it through the unit for which you wish to make a cost estimate. Estimate cost using the Y value (cost) at the point where the two lines intersect.

For example, suppose we had the following unit cost data:

Unit Number	Hours
1	3,000
2	2,400
4	1,920
8	1,536

Plotting the data on log-log paper, you will observe a straight line with an 80 percent slope.



If you extend the line-of-best-fit to Unit #100, you can estimate the cost of Unit #100. As you can see from the graph, the extended line reveals an estimated cost of approximately 680 hours for Unit #100.

7.3 Analyzing Improvement Using Lot Data and Unit Theory

Accounting System Data

Use of the improvement curve is dependent on available cost data. The relevant accounting or statistical record system must be designed to make relevant data available for analysis. Costs, such as labor-hours per unit or dollars per unit, must be identified with the unit of product.

NOTE: It is preferable to use labor-hours rather than dollars since the dollars contain an additional variable, the effect of inflation or deflation, which the labor-hours do not contain

Typically accounting systems do not record the cost of individual units. If the firm uses a job-order cost accounting, costs are accumulated on the job order in which the number of units completed are specified and costs are cut-off at the completion of the units. Process cost accounting also yields costs identified with end-item units. In this case, however, the costs are usually assigned to equivalent units produced over a period of time rather than actual units.

Average Unit Cost

To use unit improvement curve theory, you must be able to estimate the cost of a particular unit. Given lot or period costs, the only unit cost that we know is the average cost for the lot or period. However, we have a method for using average costs in improvement curve analysis.

For example, given the following data, we must be able to estimate the cost of an additional 40 units.

Lot Number	Lot Size (Units)	Lot Total Labor-Hours (Cost)	Lot Average Labor Hours (Cost)
1	6	40,800	6,800
2	9	40,500	4,500
3	15	52,500	3,500

Calculating a Lot Plot Point for Graphic Analysis

To graph the lot average unit cost, we must select a corresponding unit number. If we assume that costs go down during the lot, the average cost should occur at the middle of the lot - the lot mid-point. One problem is that the True Lot Mid-Point (the unit where the average cost is incurred) depends on the slope of the improvement curve. Unfortunately, the slope of the curve also depends on the placement of the Lot Mid-Point. The iterative process required to calculate the True Lot Mid-Point for each lot is too cumbersome for manual computation. As a result, we use the following rules of thumb for graphic analysis:

- **For All Lots After The First Lot**, calculate the lot mid-point by **dividing the number of units in the lot by two** . Then add the resulting number to all the units produced prior to the lot to determine where the unit falls in the continuing improvement curve.

For example, what would be the plot point for a lot made up of units 91 through 100. There are 10 units in the lot, so the middle of the lot would be 5 ($10 \div 2 = 5$). Adding 5 to the 90 units produced prior to the lot, we find that the plot point would be 95.

- **For a First Lot of Less Than 10**, follow the same procedure that you follow for all lots after the first lot. Of course, the lot plot point will equal the lot mid-point because no units will have been produced prior to the first lot.
- **For a First Lot of 10 or More**, calculate the lot mid-point by **dividing the number of units in the lot by three** . This adjustment is necessary to compensate for the rapid decline in cost that takes place in the first lot of production.

Given the data above, use a table similar to the following, to calculate the necessary lot plot points and lot average hours:

Lot No.	Lot Size	Cumulative Units	Lot Mid-Point	Lot Plot Point
1	6	6	3.0	3.0
2	9	15	4.5	10.5
3	15	30	7.5	22.5
You can then use this information to estimate the cost of lots that have not yet been produced. For example, suppose you wanted to estimate the cost of a Lot #4 of 40 units to be produced after the 40 units described above. The final row of the table would be:				
4	40	70	20	50
For this example, the lot plot point for Lot #4 would be at Unit #50. You would estimate the average unit cost for the lot using the cost of Unit #50.				

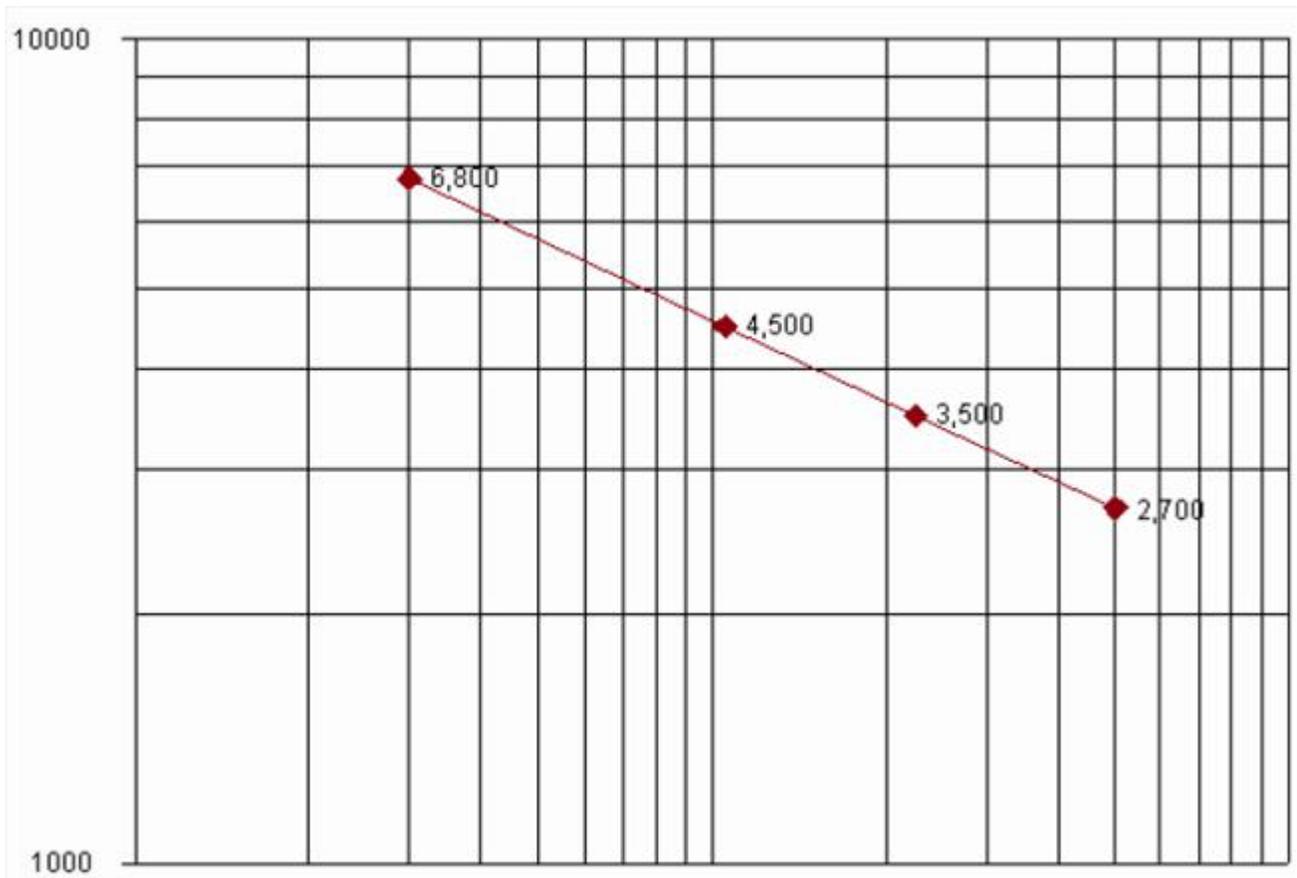
Combining Lot Plot Point and Average Unit Cost Calculation

You can combine the calculation for the lot average unit cost and the lot plot point into a single table, as shown below:

Lot No.	Lot Size	Cumulative Units	Lot Mid-Point	Lot Plot Point	Lot Average Hours	Lot Total Hours
1	6	6	3.0	3.0	6,800	40,800
2	9	15	4.5	10.5	4,500	40,500
3	15	30	7.5	22.5	3,500	52,500
4	40	70	20.0	50.0		

Plotting Data on a Log-Log Graph

Plot the average lot cost data (Y) at the corresponding lot plot point (X) on log-log paper and for an improvement curve. Extend the improvement curve through Unit #50, the lot plot point for Lot #4.



On the Y axis, the lot average cost at Unit #50 is approximately 2,700 labor hours. With this information, you can estimate the cost of Lot #4 at 108,000 labor hours (i.e., 2,700 labor hours x 40 units).

7.4 Fitting a Unit Curve

General Points to Consider

Throughout this chapter, we have assumed that all data fit a perfectly straight line. Unfortunately, most data do not fall exactly on a straight line. You need to be able to identify a trend and fit data to that trend. You can visually fit a line using graphic analysis, but most lines-of-best-fit are developed using regression analysis.

Whatever method of analysis you use to fit an improvement curve, if a data point is a significant distance away from the trend set by other data points, look into the cause of the deviation. If your analysis indicates that the data point is not comparable with the rest of the data for some reason, consider adjusting or eliminating the data point from your analysis. However, never eliminate a

data point from your analysis simply because it does not fit the apparent trend set by the remaining data.

Graphic Analysis

When visually fitting a straight line, graph the data then draw the line to minimize the distance between the straight line and the data points. Normally, you should give more weight to the larger lots as you fit the straight line.

When fitting a straight line on ordinary graph paper, you know that the line-of-best-fit must go through the average of the X values () and the average of the Y values (). When fitting a line-of-best-fit through improvement curve data on log-log paper, you have no similar fixed reference point. **Without this fixed reference point, even skilled analysts can arrive at very different lines.**

Regression Analysis

Normally, you can obtain more accurate results using regression analysis and a log-log transformation. Using the logarithmic values of X and Y instead of the actual values, the equation of the unit improvement curve ($Y = AX^B$) becomes:

$$\text{Log } Y = \text{Log } A + B(\text{Log } X)$$

The new equation describes a straight line ($Y = A + BX$) relationship. After this transformation, you can use regression analysis to fit a straight line to the data.

Improvement curve regression analysis programs differ in several ways including:

- **Use of True Lot Mid-Point.**

In addition to the accuracy gained from using regression analysis, most improvement curve programs use the true lot mid-point rather than the rule-of-thumb calculations described earlier in this section for graphic analysis. The greatest effect of using the true lot mid-point is in the first lot. Examples of the differences between the rule-of-thumb and true lot mid-points are depicted in the following table:

Selected First Lot Mid-Points				
		True-Lot Mid-Points		
Units in First Lot	Rule-of-Thumb	70% Curve	80% Curve	90% Curve
2	1.00	1.37	1.39	1.4
10	3.33	3.95	4.17	4.36

100	33.33	28.65	32.36	35.43
1,000	333.33	258.15	304.43	340.67
10,000	3333.33	2,495.48	3,002.85	3,384.18

Differences in calculating the lot mid-point will affect the results of the improvement curve analysis by the placement of the data points for analysis.

- **Method of Regression.**

Not all improvement curve analysis programs use the same mathematical model for regression analysis. For example, some analysis programs assign a weight to each lot based on the lot size, while others do not. Software using unweighted regression considers all lots (large and small) equally. When weights are assigned to each lot based on lot size, larger lots receive more analysis consideration than smaller lots.

- **Measures of Fit.**

- Regardless of the regression model used to develop the line-of-best-fit, virtually all regression analysis software will provide measures of the line's goodness of fit.
- The primary goodness of fit measure is the coefficient of determination (r^2) for the equation. As described in the chapter on "Using Regression Analysis," the coefficient of determination indicates the portion of variation in Y is explained by the regression line (e.g., an r^2 of .94 indicates that 94 percent of the variation in Y is explained by the relationship between X and Y).
- Many improvement curve analysis programs also provide the T-test for significance of the regression equation.

- **Graphic Analysis Capability.**

Many regression analysis programs provide a capability to graph the data and the regression line. For most analysts, this display is one of the strongest tools for identifying anomalies in the data that affect the value of the regression analysis as an estimating tool.

7.5 Estimating Using Unit Improvement Curve Tables

Estimating Choices

Once the cost of Unit #1, in hours or dollars, and the slope of the improvement curve have been established, we can develop estimates of future costs in several ways. You could graph the data on log-log paper and read your estimates from the graph. You could substitute the values into the improvement curve equation. Many analysts use a third choice, improvement curve tables.

Improvement Curve Tables

Improvement curve tables are an expansion of the X^B portion of the basic unit improvement curve equation, $Y = A X^B$. The result is recorded as a decimal fraction, which is typically calculated to six or eight decimal places. There is a different table value for each unit and slope. Below is an illustration of a partial improvement curve table.

Partial Improvement Curve Table						
79 Percent		80 Percent		81 Percent		
Unit	Cum Total	Unit	Cum Total	Unit	Cum Total	Unit
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	1.790000	0.790000	1.800000	0.800000	1.810000	0.810000
3	2.478245	0.688245	2.502104	0.702104	2.526065	0.716065
4	3.102345	0.624100	3.142104	0.640000	3.182165	0.656100
5	3.680837	0.578492	3.737741	0.595637	3.795233	0.613068
6	4.224550	0.543713	4.299424	0.561683	4.375245	0.580012
7	4.740494	0.515944	4.833914	0.534490	4.928703	0.553458
8	5.233533	0.493039	5.345914	0.512000	5.460144	0.531441
9	5.707214	0.473681	5.838864	0.492950	5.972892	0.512748
10	6.164223	0.457009	6.315374	0.476510	6.469477	0.496585
11	6.606656	0.442433	6.777485	0.462111	6.951880	0.482403
12	7.036189	0.429533	7.226831	0.449346	7.421690	0.469810
13	7.454188	0.417999	7.664747	0.437916	7.880206	0.458516
14	7.861784	0.407596	8.092339	0.427592	8.328507	0.448301
15	8.259928	0.398144	8.510538	0.418199	8.767503	0.438996
16	8.649429	0.389501	8.920138	0.409600	9.197970	0.430467
17	9.030982	0.381553	9.321821	0.401683	9.620576	0.422606
18	9.405190	0.374208	9.716181	0.394360	10.035902	0.415326
19	9.772580	0.367390	10.103736	0.387555	10.444457	0.408555
20	10.133617	0.361037	10.484944	0.381208	10.846691	0.402234
21	10.488713	0.355096	10.860211	0.375267	11.243003	0.396312
22	10.838235	0.349522	11.229900	0.369689	11.633750	0.390747

23	11.182513	0.344278	11.594336	0.364436	12.019252	0.385502
24	11.521844	0.339331	11.953813	0.359477	12.399798	0.380546
25	11.856497	0.334653	12.308597	0.354784	12.775651	0.375853
26	12.186716	0.330219	12.658929	0.350332	13.147049	0.371398
27	12.512724	0.326008	13.005031	0.346102	13.514210	0.367161
28	12.834725	0.322001	13.347104	0.342073	13.877334	0.363124
29	13.152906	0.318181	13.685335	0.338231	14.236605	0.359271
30	13.467440	0.314534	14.019894	0.334559	14.592192	0.355587

Unit Estimate.

To estimate the price or cost for a specific unit, you can simply multiply the cost of Unit #1 by the appropriate unit factor for the desired unit and slope.

Where:

$$Y_U = T_1 \times F_U$$

Y_U = Unit cost estimate

T_1 = Theoretical cost of Unit #1

F_U = Unit cost factor for the unit.

For example, if Unit #1 is 2,000 labor hours, what would be your estimate for Unit #20 if production is expected to follow an 80 percent improvement curve? The table value for Unit #20 and an 80 percent slope is .381208. The estimate would be 762.4 labor hours, calculated as follows:

$$\begin{aligned} Y_U &= T_1 \times F_U \\ &= 2,000 \text{ labor hours} \times .381208 \\ &= 762.4 \text{ hours} \end{aligned}$$

We need to do the same thing for Lot Data:

$$Y_L = (T_1 \times F_{C2}) - (T_1 \times F_{C1})$$

Where:

Y_L = Lot cost estimate

T_1 = Theoretical cost of Unit #1

F_{C2} = Cumulative cost factor for all production through the proposed lot

F_{C1} = Cumulative cost factor for all production prior to the proposed lot

For ease of calculation, this equation may be rewritten as:

$$Y_L = T_1 (F_{C2} - F_{C1})$$

For example, if Unit #1 is 4,000 labor hours and the improvement curve slope is 80 percent, what would be your estimate for Units #15 to #25? Your estimate should be 15,192.24 labor hours, calculated as follows:

$$\begin{aligned} Y_L &= T_1(F_{C2} - F_{C1}) \\ &= 4,000 (12.308597 - 8.092339) \\ &= 4,000 (4.216258) \\ &= 16,865.032 \text{ (rounded to 16,865 labor hours)} \end{aligned}$$

7.6 Identifying Issues and Concerns

Questions to Consider in Analysis

As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use an improvement curve.

- **Is improvement curve analysis used when the contract effort involves:**
 - A significant amount of manual labor in the contract?
 - Uninterrupted production?
 - Production of complex items?
 - No major technological change?
 - Or should involve, continuous pressure to improve?

- **Does the documentation to support the use of the improvement curve include:**
 - A statement describing the improvement curve theory used in developing the estimate?
 - A summary of related cost data for the product being purchased and any similar products?
 - A description of how available data were used in estimating the theoretical cost of Unit #1 and the slope of the curve?
 - A statement on how the improvement curve estimate was used in price or cost analysis?

Like CERs, improvement curves are a form of comparison estimate. Unless you are satisfied that the historical data provide a valid base for the use of an improvement curve, estimates based on the curve should be suspect.

- **Was improvement curve theory properly applied to the available data?**

Verify the application of the improvement curve to the data available. Remember that different improvement curve models will produce different results.

For instance, you may find that a unit curve will provide more reasonable results than a cumulative average curve provided by an offeror. Examine the results of both curves when an offeror proposes using a cumulative average curve, because cumulative average curves often conceal significant fluctuations in per unit labor hours.

- **Did any improvement curve analysis isolate costs associated with contract changes and production interruptions?**

Changes and production interruptions will both have a disruptive effect on improvement. If their effects are not identified and considered in analysis, improvement curve estimates will typically underestimate actual requirements. Random fluctuations around an improvement curve line-of-best-fit should be expected. However, if costs increase or decrease dramatically, you should suspect that the actual costs have been affected by a contract change or a break in production. When you suspect that actual costs are affected by a contract change or break in production, contact the cognizant auditor and Government technical personnel for assistance in your analysis.

On the other hand, an offeror might overstate the impact of an interruption in production—contending that the interruption has been so long that it will have to start from scratch. However, improvements in unit costs result in part from such factors as better product design, tooling, work methods, and work layout. If these were properly documented, some of the improvement should carry over to the new effort—regardless of the length of the interruption or turnover of personnel.

- **Does the improvement curve analysis project continued improvement?**

Occasionally, an offeror will propose "negative learning." In other words, as more units are produced, the cost per unit increases. Do not accept the negative learning argument. If something has significantly changed, consider starting a new curve with a new first unit value and slope.

- **Does the improvement curve estimate include the costs of rework and repair?**

The effort for rework and repair may or may not be included in the costs projected with the improvement curve. Therefore, you need to determine if these costs are included in the projected costs before allowing any add-on factors for rework or repair.

- 8.0 - [Chapter Introduction](#)
- 8.1 - [Identifying Situations For Use](#)
- 8.2 - [Identifying Elements Of A Labor Standard](#)
- 8.3 - [Measuring And Projecting Operation Efficiency](#)
- 8.4 - [Identifying Issues And Concerns](#)

8.0 - Chapter Introduction

In this chapter, you will learn about work measurement concepts and their application to cost analysis.

Work Measurement . Work Measurement involves the use of labor standards to measure and control the time required to perform a particular task or group of tasks. Most often labor standards are developed and applied in manufacturing operations; however labor standards can be used in estimating and managing the cost of a vast variety of activities including engineering drafting, clerical administration, and janitorial services.

Work Measurement System . A Work Measurement System is a management system designed to:

- Analyze the touch labor content of an operation;
- Establish labor standards for that operation;
- Measure and analyze variances from those standards; and
- Continuously improve both the operation and the labor standards used in that operation.

Work Measurement System Plan . A Work Measurement System Plan is the firm's program for implementing, operating, and maintaining work measurement in its operations. As a minimum, the plan should provide guidance on:

- Establishing and maintaining standard accuracy;
- Conducting engineering analyses to improve operations;
- Revising standards and related system data; and
- Using labor standards as an input to budgeting, estimating, production planning, and performance evaluation.

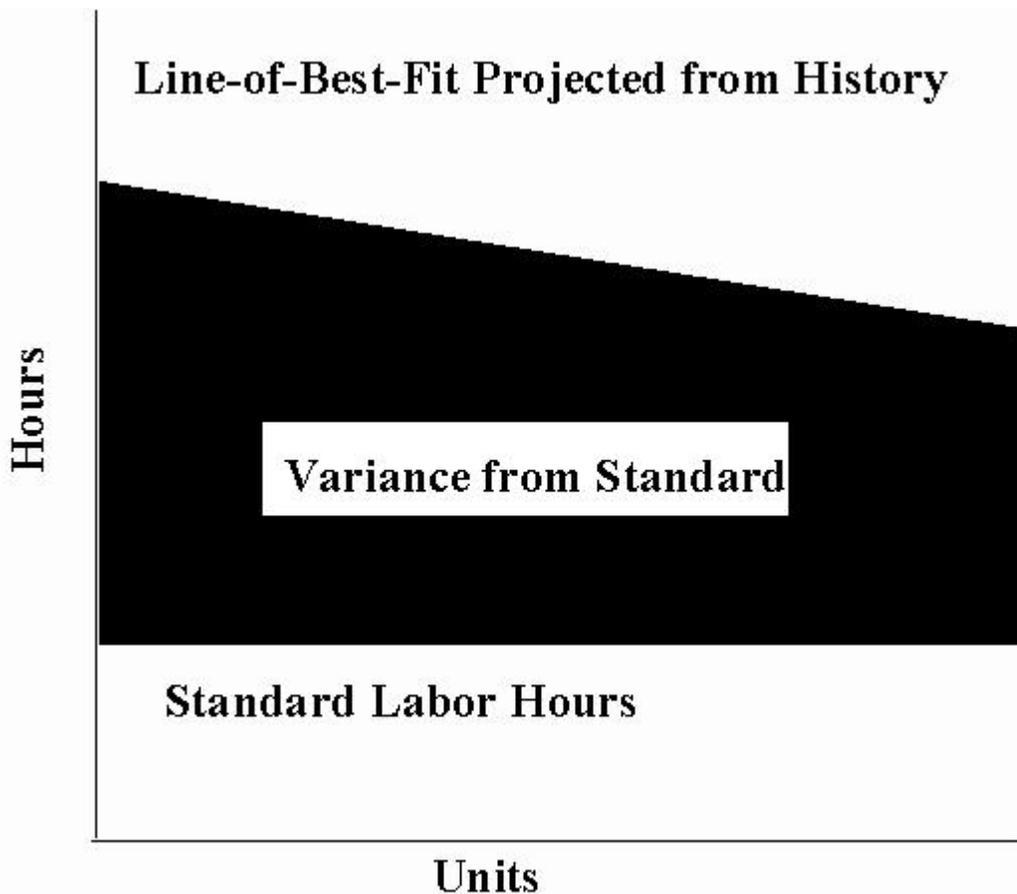
Labor Standard Types . A labor standard is a measure of the time it should take for a qualified worker to perform a particular operation. Labor standards are commonly grouped into two types:

- **Engineered standards** are developed using recognized principles of industrial engineering and work measurement. The standards developed define the time necessary for a qualified worker, working at a pace ordinarily used, under capable supervision, and experiencing normal fatigue and delays, to do a defined amount of work of specified quality when following the prescribed method. As a result, you can use engineered standards to examine contractor estimated labor hours and to identify any projected contractor variances from that estimate.
- **Non-engineered standards** are developed using the best information available without performing the detailed analysis required to develop engineered standards. Historical

costs are commonly used standards that typically measure the hours that have been required to complete a task rather than the hours that should be required.

Estimate of Efficient Operation Cost . Standards provide information on what it should cost to complete an operation or series of operations in product production. Instead of applying pressure to improve in all areas, managers can use this information to identify areas requiring particular management emphasis. The Acquisition Team can use that same information to identify inefficient operations for close scrutiny during contract negotiations.

- The log-log graph below presents a line-of-best-fit developed using actual labor-hour history. Note that this line follows the form of the improvement curve. Without labor standards, the firm and the Government would likely project the improvement curve to estimate the labor hours required to produce future units.



- Labor standards provide additional information that can be used in estimate development and analysis. The vertical distance between the labor-hour history and the labor standard represents the variance from the standard. Some of that variance may be related to inefficiencies that cannot be resolved. However, all elements should be targeted for identification and analysis. Key elements include:

- Technical factors (e.g., manufacturing coordination, engineering design changes, fit problems, design errors, operation sheet errors, tooling errors, work sequence errors, and engineering liaison problems).
- Logistics (e.g., incorrect hardware and parts shortages).
- Miscellaneous factors (e.g., unusual working conditions, excessive overtime, and excessive fatigue).
- Worker learning (e.g., familiarity with processes and methods).
- Variance analysis should identify, categorize, and develop plans to control all variances from standard. Plans will typically concentrate on the operations with the largest variances from standard, because these operations present the greatest opportunity for cost reduction.

Updating Standards . Standards cannot be set and forgotten. Process improvement is one of the central elements of an effective Work Measurement System. As methods improve, the associated labor standards must be updated.

Standards changes will affect the estimating value of all the data based on those standards. For example, some variance analyses may remain valid while other analyses will be rendered meaningless as a result of the change. The system must assure that valid analyses are retained for continued utilization. At the same time, the system must also assure that meaningless data are not misused.

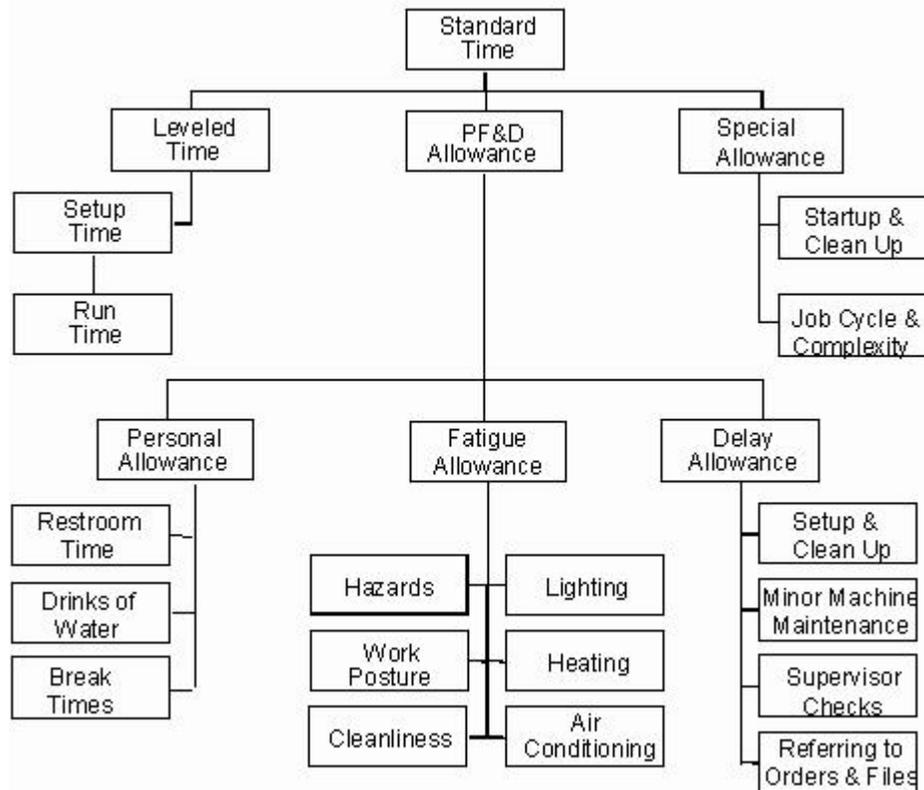
8.1 - Identifying Situations For Use

General Situations . Contractors should consider the use of labor standards whenever contractor employees will be performing the same tasks repetitively over an extended period of time. Labor standard development requires extensive detailed effort. The time and cost required for standards development are prohibitive unless the task will be performed repetitively. On the other hand, when an operation will be performed repetitively, the cost visibility provided by labor standards permits detailed cost evaluation and control that can result in significant savings to the Government. To be of real value, labor standards must be considered in making key management decisions (e.g., budgeting, estimating, production planning, and performance evaluation).

8.2 Identifying Elements Of A Labor Standard

Key Elements . As a contracting officer, it is likely that you will never be required to develop a labor standard. However, you may be called upon to negotiate a contract price based, in part, on labor standards. Therefore, you should know the elements of a standard and how they are developed.

The figure below depicts some of the factors that should be considered in each element.



Leveled Time . Leveled time is the time that a worker of average skill, making an average effort, under average conditions, would take to complete the required task. There are a variety of techniques used in leveled time development, but the four used most commonly are:

- **Time Study.** In performing a time study, industrial engineers (or other labor analysts) time the effort required to perform a defined task. While it may sound simple, this is a complex process that requires special training and experience. To perform a time study, the analyst must:
 - Clearly define and document the work design, including the best design of the work place, tools, tasks, and subtasks.
 - Select a person to be timed. The person selected should be receptive to being timed, experienced in the work methods being used, and familiar with the tasks and subtasks of the work design.
 - Observe and record the time that the selected worker requires to perform each of the subtasks in the work design. Several observations are required to average out random variations and assure that all elements of the work have been considered. The number of observations required will increase as the confidence level desired by the analyst increases and as the variability between observed times increases.

- Assign a pace rating based on an evaluation of how the ability and effort of the worker being timed compares with those of an average worker. Using the pace ratings, the analyst converts observed times into a leveled time for the subtask.
 - Total subtask times to develop a leveled time for the entire task.
- **Predetermined Leveled Times.** Instead of using time study to develop a leveled time, the analyst can use predetermined leveled times (also called predetermined standards or basic motion standard data). Predetermined leveled times are established for basic body motions, such as reach, move, turn, grasp, position, release, disengage, and apply pressure. The analyst may obtain them from published standards in tabular or electronic forms, or the firm may develop its own. To use predetermined leveled times, the analyst must:
 - Clearly define and document the work design, including the best design of the work place, tools, tasks, and subtasks.
 - Select and document the source of the predetermined leveled times.
 - Identify and document the basic body motions involved in performing each subtask. Motions for each hand must be specifically identified. The need for precise measurement of complex body motions for each job element may make this method of leveled time development inappropriate for complex tasks with long performance cycle times.
 - Assign times to the body motions required to complete each subtask and total assigned times to develop a leveled time for the subtask. Documentation should demonstrate that the accuracy of the original data base has not been compromised in application or standard development.
 - Total subtask times to develop a leveled time for the entire task.
- **Standard Time Data.** Standard time data (or elemental standard data) are developed for groups of motions that are commonly performed together, such as drilling a hole or painting a square foot of surface area. Standard time data can be developed using time studies or predetermined leveled times. After development, the analyst can use the standard time data instead of developing an estimate for the group of motions each time they occur.
 - Typically, the use of standard time data improves accuracy because the standard deviations for groups of motions tend to be smaller than those for individual basic motions. In addition, their use speeds standard development by reducing the number of calculations required.
 - Estimate development using standard time data is much like using predetermined leveled times except that groups of motions are estimated as a single element instead of individual body motions.
- **Work Sampling.** Work sampling is commonly used to develop non-engineered standards. It cannot be used alone to develop engineered standards. However, it can be used to supplement or check standard development by more the definitive techniques described above. For example, it can be used to determine job content and assess productive vs. nonproductive time.
 - In work sampling, analysis is based on a large number of random, rather than continuous observations. Estimates are based the proportion of time spent by one or more persons on a given activity. This is useful for jobs with irregular components that vary in the amount of time per unit of output.

- To use work sampling in standard development, the analyst must:
 - Identify and define activities involved in the work (through discussions with the workers and preliminary observations).
 - Develop the method(s) for observing and recording activities.
 - Determine the sampling strategy (e.g., stratified) and number of observations (by time and place).
 - Record observed activities during each period.
 - Consolidate and analyze the data.
 - Use the data collected to develop nonengineered standards or to supplement development of engineered standards.

PF&D Allowance . After the leveled time is developed, estimators must consider a personal, fatigue, and delay (PF&D) allowance. Be careful when contractors use predetermined time systems. Some predetermined time systems include a partial or complete allowance for PF&D. If the contractor uses such standards, additional PF&D consideration may not be appropriate.

Contractor work measurement policies and procedures should provide the detailed rationale used for applying PF&D allowances. Each allowance should be identified and quantified. Total PF&D allowances typically total 15 percent. However, allowances may be higher or lower depending on the nature of the work and related working conditions. For example, strenuous work in an extremely hot environment would typically merit a higher PF&D allowance than light labor performed in an air conditioned room.

Personal Allowance. A personal allowance considers time for a worker to take care of personal needs, such as trips to the rest room and drinking fountain. The table below

Personal Allowance Considerations		
Personal allowance documentation should document:	Considerations	Typical Percentage Allowance
A Basic Allowance which considers the breaks available for work during an 8-hour day.	Most firms allow, at least, two 10-minute breaks during each 8-hour shift, the basic personal allowance is 4.2 percent (20 minutes/480 minutes).	4.2
	Normal office conditions	0.0
	Normal shop, central heat, slightly dirty or greasy.	1.0
	Slightly disagreeable conditions. Exposed to inclement weather part of the time, poor heating, or poor cooling.	3.0
	Extremely disagreeable conditions. Proximity to hot objects, continuous	6.0

	exposure to disagreeable odors and fumes, or to excessive temperature ranges.	
	Total time allowed:	
	5 minutes	1.0
	10 minutes	2.1
	15 minutes 20 minutes	3.1 4.2
Any allowance for work performed in a super-clean room.	An additional allowance may be added to consider the time require to assure that super- clean room requirements are met.	4.0

Fatigue Allowance. A fatigue allowance considers the time required to recuperate from fatigue.

Fatigue Allowance Considerations		
Personal allowance documentation should document:	Considerations	Typical Percentage Allowance
Any allowance for handling heavy weights.	Effective Net Pounds Percent of Time Under Load Handled 1-12 13-25 26-50 51-75 76-100 1-10 0 1 2 3 4 11-20 1 3 5 7 10 21-30 2 4 9 13 17 31-40 3 6 13 19 25 41-50 5 9 17 25 34 51-60 6 11 22 x x 61-70 7 14 28 x x 71-80 8 17 34 x x x - Study for possibilities for worker rotation and other means to relieve fatigue.	Select percentage from table.
	Multiply the table values above by the following factors to consider lifting requirements: For picking up from the floor, multiply the table value by 1.10. For placing the load above chest height, multiply table value by 1.20. For getting the load from chest height, multiply the basic allowance by 0.50.	Depends on work.
	For sliding and rolling objects, multiply the weight by the coefficient of friction to determine the	Depends on work.

	<p>effective weight moved.</p> <p>Coefficients of Friction (Average Values)</p> <p>Surfaces Friction Coefficient</p> <p>Wood on Wood 0.4</p> <p>Wood on Metal 0.4</p> <p>Metal on Metal 0.3</p>	
	Sitting or standing. (Work will normally be less tiresome if the position is varied frequently.)	0.0
	Sitting.	1.0
	Walking.	1.0
	Standing.	2.0
	Climbing or descending ramps, stairs, or ladder.	4.0
	Working in close cramped quarters.	7.0
Any allowance for the mental requirements of the job.	Work largely committed to habit (e.g., simple calculations on paper, reading easily understood material, counting and recording, simple inspection requiring attention but little discretion, or arranging papers by letter or number).	0.0
	Work requires full attention (e.g., copying numbers or instructions, remembering part number while checking a parts list, or filing papers by subject of familiar nature).	2.0
	Work requires concentrated attention (e.g., reading of nonroutine instructions or cross-checking items).	4.0
	Work requires deep concentration (e.g., making swift mental calculations or memorizing items).	8.0
Any allowance for the lighting on the job site.	<p>Continual glare on work area.</p> <p>Work requiring constant change of light.</p> <p>Less than 75 foot candle power on work surface for normal work.</p> <p>Less than 125 foot candle power on work surface for close work.</p>	2.0
Any allowance for noise on the job site.	Constant, rather loud noises over 60 decibels (e.g., machine shops or motor test shops).	1.0
	Average constant noises, level but with loud, sharp, intermittent, or staccato noise (e.g., nearby riveters or punch presses).	2.0

	0.00 to 0.20 minute cycles	4.0
	0.21 to 0.40 minute cycles	3.0
	0.41 to 0.80 minute cycles	2.0
	0.81 to 2.50 minute cycles	1.0
	2.51 minutes or more	0.0
Any allowance for the use of safety devices or clothing.	No allowance should be made here unless it is necessary to remove the equipment occasionally for relief or if wearing the item causes fatigue.	
	Face shield	2.0
	Rubber boots	2.0
	Goggles or welding mask	3.0
	Tight, heavy protective clothing	4.0
	Filter mask	5.0
	Safety glasses	0.0

Delay Allowance. A delay allowance covers unavoidable, predictable and unpredictable delays for such activities as replenishing materials, rejecting nonstandard parts, making minor equipment repairs, and receiving instructions.

Delay Allowance Considerations		
Personal allowance documentation should document:	Considerations	Typical Percentage Allowance
Basic Allowance	Isolated job. Little coordination with adjacent jobs.	1.0
	Fairly close coordination with adjacent jobs.	2.0
	Worker moves once each 5 minutes.	5.0
	Worker moves once each 30 minutes.	3.0
	Worker moves once each 60 minutes.	2.0
	Worker moves once each 2 hours.	0.0

Special Allowances . Any proposed special allowance must be supported by detailed engineering analysis. An appropriate study should be conducted in each shop or functional area to ascertain any requirement for a separate delay allowance. The analyst should assure that there is no duplication between cycle time elements and allowance elements and that the Special

Allowance does not become a dumping ground for operation activity that is not an integral part of shop work load.

- Work elements such as cleaning chips from equipment, tool care, or tool replacement, though occurring irregularly, should be measured and the time required prorated directly to the machine operating portion of the work cycle rather than as an allowance.
- Certain other irregularly occurring elements having a direct relationship to the job such as obtaining parts and materials and periodic inspection should be added to the cycle time on a prorated basis or as a separate work element rather than added as an allowance.

When a special allowance is appropriate, the time required is first calculated in minutes and then converted to a percentage. The base for calculating and applying the allowance percentage is normally the sum of the leveled time and the PF&D allowance. Appropriate special allowances typically fall into two categories:

- Those that consider elements that occur on an unforeseeable basis:
 - Power failures of nonreportable duration.
 - Minor repairs to defective parts.
 - Waiting for a job assignment.
 - Obtaining job information from a supervisor, inspector, or production control specialist.
 - Unsuccessful hunt for parts or materials.
 - Machine breakdown of nonreportable duration.
- Those that consider elements that occur periodically (daily, weekly, hourly) such as:
 - Cleaning and lubricating equipment.
 - Work area clean-up.

Applying an Allowance to Leveled Time . Allowances are normally expressed as a percentage of standard time spent unproductively (e.g., a 15 percent PF&D Allowance indicates that 15 percent of the worker's standard time is spent unproductively). To apply an allowance, the analyst must determine how much the leveled time must be increased to allow for the unproductive time. This is accomplished by dividing the leveled time by the percentage of time spent productively.

$$T_s = \frac{T_L}{1.00 - A_{PF\&D}}$$

Where:

T_s = Standard time

T_L = Leveled time

$A_{PF\&D}$ = PF&D allowance in decimal form

For example: The leveled time for a particular task is 170 minutes, the PF&D Allowance is 15 percent, and there is no special allowance. The standard time would be calculated as:

$$\begin{aligned}
T_s &= \frac{T_L}{1.00 - A_{PF\&D}} \\
&= \frac{170}{1.00 - .15} \\
&= \frac{170}{.85} \\
&= 200 \text{ minutes}
\end{aligned}$$

Note: The leveled time is 85 percent of the standard time (85% of 200 is 170). The remaining 15 percent of the standard time (15% of 200 minutes is 30 minutes) is the allowance for personal, fatigue, and delay factors.

8.3 - Measuring And Projecting Operation Efficiency

Comparing Labor Standard with the Actual Time . Standards represent goals for efficient operation. Tasks are rarely completed in the allowed standard time. Work Measurement Systems commonly use realization or efficiency factors to evaluate how the actual time required to complete a task compares with the standard time for that task. Analysts can then use these measures to identify tasks that require special analysis to identify and correct inefficient operations.

Since estimators strive to estimate realistic contract costs, they use realization or efficiency factors with labor standards to estimate future labor hours required to complete the task.

Calculating a Realization Factor . A realization factor is generally a measure of overall performance (e.g., shop, product line, or plant). It will normally be calculated from historical data as:

$$F_R = \frac{T_A}{T_s \times R}$$

Where:

F_R = Realization factor

T_A = Actual time to perform the work

T_s = Standard hours for the task

R = Repetitions of the task included in the work

Don't be confused by the fact that some firms refer to this calculation as an efficiency factor.

For example: A task has a standard time of 1.5 hours. Actual time to perform the task 100 times is 300 hours. Using the model at ***, the realization factor would be 2.00

$$\begin{aligned}
F_R &= \frac{T_A}{T_S \times R} \\
&= \frac{300}{1.5 \times 100} \\
&= \frac{300}{150} \\
&= 2.00
\end{aligned}$$

In the above example, actual experience shows that the work takes twice as many hours as the standard time indicates.

Developing an Estimate Using a Realization Factor . The estimator can use the standard time and realization factor to develop a realistic labor-hour estimate using the model at ***.

For example. An estimate of the actual time to complete the task above for 50 units would be calculated as:

$$\begin{aligned}
Y &= T_S \times R \times F_R \\
&= 1.5 \times 50 \times 2.00 \\
&= 150 \text{ labor hours}
\end{aligned}$$

Where:

Y = Estimated hours

All other symbols are as defined above

Calculating an Efficiency Factor . An efficiency factor is calculated to demonstrate efficiency against the standard (e.g., a task with an efficiency factor of .60 is being performed at 60 percent efficiency). The factor is normally calculated:

$$F_E = \frac{T_S \times R}{T_A}$$

Where:

F_E = Efficiency factor

All other symbols are as defined above

For example. A task has a standard of 1.8 hours. Actual time to perform the task 100 times is 400 hours. The efficiency factor would be calculated as follows:

$$\begin{aligned}
 F_E &= \frac{T_S \times R}{T_A} \\
 &= \frac{1.8 \times 100}{400} \\
 &= \frac{180}{400} \\
 &= .45 \text{ or } 45 \text{ percent efficiency}
 \end{aligned}$$

Developing an Estimate Using an Efficiency Factor . The estimator can use the standard time and efficiency factor to develop a realistic labor-hour estimate.

For example. An estimate of the time to complete the task above for 50 units would be calculated as:

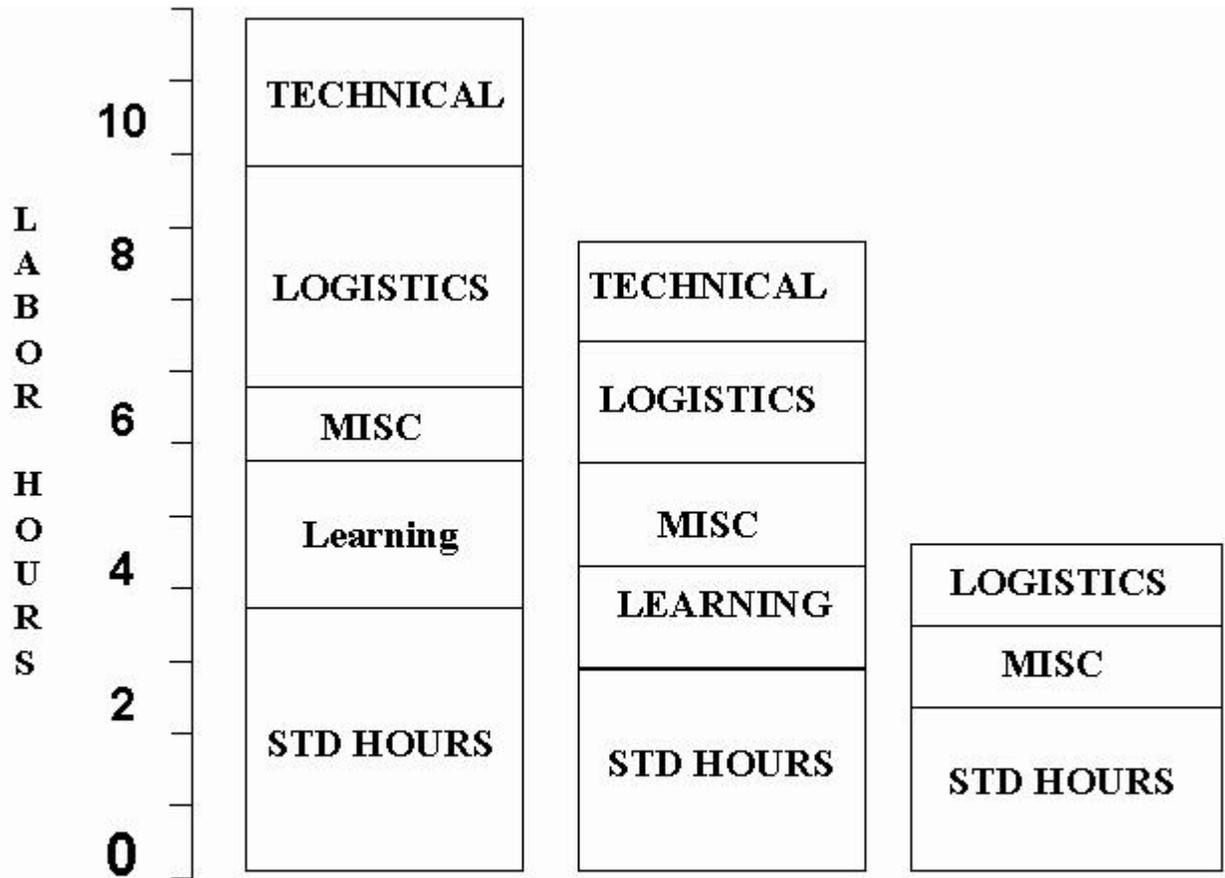
$$\begin{aligned}
 Y &= \frac{T_S \times R}{F_E} \\
 &= \frac{1.8 \times 50}{.45} \\
 &= 200 \text{ labor hours}
 \end{aligned}$$

Analyzing Realization and Efficiency Factors . Analysis of labor estimates developed using labor standards requires extensive knowledge and experience. Even skilled industrial engineers typically require special training in work measurement analysis. As a result, you should normally request technical support whenever an offeror estimates labor hours using labor standards.

For each standard, offerors should be required to provide information on internal analyses of the variance between the actual time required to complete the work and the standard time to determine the causes for the variance and identify ways of managing performance improvement.

You should expect offeror's to demonstrate continued improvement in realization and efficiency factors. The figure below depicts some of the reasons for that improvement.

- At Unit #1, total labor-hours include substantial inefficiencies related to technical, logistics, learning, and other factors.
- As production increases, there should be reductions in all areas of inefficiency. In most cases, there should also be an improvement in the labor standard itself, as better production methods are identified and implemented.
- By Unit #1000, the contractor should be operating efficiently, with only minor inefficiencies related to such factors as unavoidable parts shortages.



Typical Variances from Standard during a Production Program

However, improvement will not automatically follow this pattern. Effective analysis and management effort are required. Even when these are present, improvement may be hampered by factors such as repeated changes in design or production methods.

Still, the goal of both the contractor and the Government should be continuous improvement. Even when operations are being performed at or close to standard, the contractor should be searching for methods improvements that will reduce costs and improve overall efficiency.

Projecting Realization and Efficiency Factors . Be cautious of any estimate for continuous production that does not consider variance reduction following the improvement curve. Continuous improvement is one of the reasons for using labor standards, because standards provide detailed information on the areas that offer the greatest opportunity for improvement.

Improvement curves and moving averages are commonly used to project variation from labor standards. Either technique can be acceptable depending on the situation. Technical assistance can be very valuable in evaluating offeror forecasts.

- **Improvement Curves.** Using an improvement curve to track and project variance from a labor standard assumes that the variance is related to the number of units produced. As more units are produced, the variance is expected to decline following improvement curve theory.
 - **Moving Averages.** Firm's often use moving averages to track and project variance from a labor standard when the variance is not expected to follow an improvement curve. This assumption is often valid when product production is not continuous or there are frequent changes in design or production methods.
 - While moving averages are an acceptable way to track and project variances, do not forget that a firm could be using a moving average to hide a downward trend in the data. That is particularly true in cases where the firm proposes a single moving average calculated over a large number of periods.
 - Government technical personnel can provide invaluable assistance in assuring that averages are not masking a trend in the data and that data from one or two periods are not unduly affecting the average.
-

8.4 Identifying Issues And Concerns

Questions to Consider in Analysis . As you perform cost analysis, consider the issues and concerns identified in this section, whenever you use work measurement.

- *Is the offeror using available standards and realization or efficiency factors to estimate contract cost?*

If the offeror has an active Work Measurement System, the System provides vital insight into direct labor costs. Information on related time standards and variance analysis are cost or pricing data and must be provided by the offeror whenever you require cost or pricing data. You should also consider requesting information on time standards and variance analysis, whenever you require an offeror to submit information other than cost or pricing data.

- *Were standards developed using appropriate process analysis and accepted methods of standard development?*

Many firms refer to historical costs as standards. Using historical costs does not provide the methods analysis and engineering discipline normally associated with the use of engineered labor standards in estimating.

- *Are realization or efficiency factors based on experience with the same or similar products and processes?*

In a cost proposal, either factor should consider experience with the same product or similar products and processes. A realization standard at one facility should not be used at another facility unless the conditions are the same. Do not compare realization factors for one facility with those for other facilities.

- *Are standards and factors current?*

The data used to develop standards should be current and representative of current methods, facilities, and working conditions. Realization/efficiency factors should be based on the current standards. If you have questions or concerns, seek assistance from Government technical and audit representatives.

- *What efforts are being taken to control variance from labor standards?*

Reasons for the differences between the standard hours and actual hours should be explained. Improvement curves are often used to estimate the reduction of variances from standard as production continues. Setting and achieving aggressive goals for improvement of realization or efficiency factors beyond historical improvement curve effects should be a prime factor in reviewing contractor performance.

- *How are rework and repair considered in the estimate?*

Rework and repair occur when a part or assembly is rejected in an inspection or test and sent back for correction of the deficiency. In addition, some completed parts and assemblies must be reworked to incorporate design changes. The cost of rework should not be included in the labor standard, related allowances, or the realization factor. Instead, time spent on rework should be accounted for separately. However, labor standards can be used in estimating the labor effort required for rework. Historical rework costs should be carefully screened to eliminate rework costs associated with one-time problems or changes.

- *Is the method used for realization or efficiency factor forecasting appropriate?*

Use of improvement curves is appropriate in continuous production situations that should foster variance reduction. Use of moving averages is appropriate in situations where sporadic production or other factors hamper efforts to reduce any variance from standard.

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- 9.2 - [Selecting A Discount Rate](#)
- 9.3 - [Identifying Cash Flows To Consider](#)
- 9.4 - [Identifying Cash Flows To Consider](#)
 - 9.4.1 - [Evaluating Your Market Research](#)
 - 9.4.2 - [Developing Your Price Estimate](#)
- 9.5 - [Calculating Net Present Value And Selecting The Best Alternative](#)
- 9.6 - [Identifying Issues And Concerns](#)

9.0 - Chapter Introduction

In this chapter, you will learn to use net present value analysis in cost and price analysis.

Time Value of Money . The time value of money is probably the single most important concept in financial analysis. When we say that money has time value, we mean that a dollar to be received today is worth more than a dollar to be received at any future time. Money has a time value because of the opportunity to earn interest or the cost of paying interest on borrowed capital.

For example, assume that you need to buy a new car but do not have the money that you need to pay for it. You must borrow the entire purchase price. Two dealers offer to sell you identical cars for \$21,000. Dealer #1 requires cash on delivery. Dealer #2 will provide you an interest-free loan for one year. Where would you buy the car? Probably from Dealer #2, because you will save all the interest for the first year of ownership.

Present Value . In the example above, Dealer #2 was clearly the low-cost choice (because of the interest-free loan for one year), but what if Dealer #1 offered the car at a lower price, say \$20,000? Which would be the low-cost choice then?

To make that decision, you must be able to determine the present value of each alternative. If you could invest \$20,000 at 5.0 percent interest, it would be worth \$21,000 at the end of one year. Based on that calculation, we could say that \$20,000 is the present value of \$21,000 one year from now when the interest rate is 5.0 percent. At that interest rate, you would presumably be indifferent about where to buy your car because the present value of the two choices is the same.

Net Present Value . Calculating present value may involve receipts as well as expenditures. For example, the alternatives may have some salvage value after their useful life has ended. The estimated receipt from the sale of the item must be considered in your analysis. The difference between the present value of the receipts and the present value of the expenditures is net present value. The best financial choice is the alternative with the highest net present value. In procurement, the alternative with the highest net present value is the alternative with the smallest payment net present value.

Factors Affecting Net Present Value . The major factors affecting present value are the timing of the expenditure (receipt) and the discount (interest) rate. The higher the discount rate, the lower the present value of an expenditure at a specified time in the future. For example, as you learned above, \$20,000 is the present value of \$21,000 one year from now when the interest rate is 5.0 percent. If the interest rate were 10.0 percent, \$19,091 would be the approximate present value of \$21,000 one year from now.

Note that the change in the interest rate would have a significant affect on your net present value analysis in the car case. Your choice is still to pay \$20,000 now or \$21,000 a year from now. At an interest rate of 10 percent the present value of \$21,000 to be paid a year from now is \$19,090.90, which is less than the \$20,000 offer. So it appears that the low-cost choice is to wait and pay the \$21,000 in one year.

Office of Management and Budget (OMB) delineates the rates that you should use in Government net present value analysis. These rates are based on the rate that the Treasury Department pays to borrow money for periods from 91 days to 30 years. These rates are updated

annually at the time of the President's budget submission to Congress. These rates can be found in [Appendix C - OMB Circular A-94, Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs](#) , or obtained by telephoning (202) 395-3381.

Net Present Value Analysis . Regardless of the application, you should use this 5-step process in net present value analysis:

Step 1. Select the discount rate.

Step 2. Identify the costs/benefits to be considered in analysis.

Step 3. Establish the timing of the costs/benefits.

Step 4. Calculate net present value of each alternative.

Step 5. Select the offer with the best net present value.

Lease-Purchase Analysis Examples (OMB Circular A-94, Paragraph 13). In this chapter, we will demonstrate the application of net present value analysis concepts using lease-purchase examples. Our use of these examples is not meant to ignore other uses of net present value analysis in Government contracting.

9.1 - Identifying Situations For Use

OMB Suggested Use (FAR 23.203 and OMB Circular A-94). Unless precluded by agency procedures, OMB Circular A-94 suggests the use of net present value analysis in any analysis to support Government decisions to initiate, renew, or expand programs or projects which would result in a series of measurable benefits or costs extending for three or more years into the future. Examples of acquisition decisions that involve such analyses include:

- Lease-purchase analyses;
- Analyses of different lease alternatives;
- Life-cycle cost analyses; and
- Trade-off analyses considering acquisition costs and energy-utilization costs of operation.

Required Lease-Purchase Analysis (OMB Circular A-94, Paragraph 13). In addition to the suggested application to any benefit-cost analysis, OMB Circular A-94 requires that any decision to lease a capital asset be justified as preferable to direct Government purchase and ownership in situations where both of the following are true:

- The lease-purchase analysis concerns a capital asset or a group of related assets whose total fair market value exceeds \$1 million.
- The lease-purchase analysis concerns a capital asset (including durable goods, equipment, buildings, facilities, installations, or land) which is:

- Leased to the Government for a term of three or more years;
- New, with an economic life of less than three years, and leased to the Government for a term of 75 percent or more of the economic life of the asset;
- Built for the express purpose of being leased to the Government; or
- Leased to the Government and clearly has no alternative commercial use (e.g., a special-purpose Government installation).

The analysis conducted in support of that justification should involve net present value analysis and can be performed in one of three ways, as delineated in the table below:

Methods of Lease-Purchase Analysis	Use...
Conduct a separate lease-purchase analysis for each acquisition.	<p>Only for major acquisitions. A lease is a major acquisition when one of the following is true:</p> <ul style="list-style-type: none"> • Acquisition is a separate line item in the Agency's budget. • The agency or the OMB determines that the acquisition is a major one. • The total purchase price of the asset or group of assets will exceed \$500,000.
Conduct periodic lease-purchase analysis of the recurring acquisition of assets for the same general purpose.	For an entire class of assets.
Adopting a policy for smaller leases and submitting the policy to OMB for approval.	<p>Normally after the agency demonstrates that:</p> <ul style="list-style-type: none"> • The leases in question would generally result in substantial savings to the Government. • The leases in question are so small or so short as to make separate analyses impractical. • Leases of different types are scored consistent with the requirements of OMB Circular A-11, Preparation and Submission of Annual Budget Estimates.

9.2 - Selecting A Discount Rate

OMB Discount Rate Guidance (OMB Circular A-94, Appendix C). Unless precluded by agency practice, you should use the current discount rates contained in OMB Circular A-94, Appendix C.

Nominal Treasury Rates (OMB Circular A-94, Appendix A & Appendix C). For most benefit-cost analysis you should use nominal discount rates (i.e., discount rates that include the effect of actual or expected inflation/deflation). *Real Treasury Rate* (OMB Circular A-94, Appendix A & Appendix C). For some projects (e.g., long-term real estate leases), you may find it more reasonable to state payments in terms of stable purchasing power (that is, constant dollars) and adjust them separately using a pre-determined price index. In such situations, cash flows should be discounted using the real Treasury borrowing rate for debt of comparable maturity. The real Treasury rate is the nominal Treasury rate adjusted to eliminate the effect of anticipated inflation/deflation. These rates are also contained in OMB Circular A-94, Appendix C and are updated annually.

Selecting the Rate for Analysis . Whether you are using nominal or real treasury rates, match the rate to the analysis period (e.g., use 5.6 percent to discount all expenditures/ receipts for a 3-year lease analysis).

To analyze a project requiring analysis for a period different from those presented above, use linear interpolation to determine the appropriate discount rate.

Step 1. Estimate the change in the discount rate for each year between the next lower and next higher maturity period with identified discount rates.

$$R_c = \frac{M_2 - M_1}{R_2 - R_1}$$

Where:

R_c = Change in the discount rate for each year difference in the project maturity period

M_1 = Next lower maturity period with an identified discount rate

M_2 = Next higher maturity period with an identified discount rate

R_1 = Discount rate for maturity period M_1

R_2 = Discount rate for maturity period M_2

Step 2. Calculate the interpolated rate using the rate for the next lower maturity period with an identified rate and the estimated change in the discount rate for each year difference in the maturity period.

$$R_I = R_1 + R_c (M_P - M_1)$$

Where:

R_I = Interpolated discount rate for the project maturity period

M_P = Maturity period for the project

All other symbols are as defined above

Linear Rate Interpolation Example. The following example demonstrates the steps involved in interpolating a nominal interest rate for evaluating an 8-year lease:

Step 1. Estimate the change in the discount rate for each year between the next lower and next higher maturity period with identified discount rates.

$$\begin{aligned} R_c &= \frac{M_2 - M_1}{R_2 - R_1} \\ &= \frac{5.9 - 5.8}{10 - 7} \\ &= \frac{.1}{3} \\ &= .033 \end{aligned}$$

Step 2. Calculate the interpolated rate using the rate for the next lower maturity period with an identified rate and the estimated change in the discount rate for each year difference in the maturity period.

$$\begin{aligned} R_I &= R_1 + R_c (M_P - M_1) \\ &= 5.8 + .033 (8 - 7) \\ &= 5.8 + .033 (1) \\ &= 5.8 + .033 \\ &= 5.833 \end{aligned}$$

9.3 - Identifying Cash Flows To Consider

Cash Flow. A cash flow is a receipt or expenditure related to the proposed lease or purchase. Guidance on the costs/benefits that you should consider in lease-purchase analysis is provided in both FAR and OMB Circular A-94. The solicitation should require each offeror to identify relevant cash flows associated with its proposal. Remember, the purpose of the Government evaluation is to identify the best net present value.

Analysis Period (OMB Circular A-94, Paragraph 13c8). In lease-purchase analysis, the proper period for analysis is the lease period including all renewal options. The period of the projected lease must be defined in the solicitation to assure identification and analysis of all relevant cash flows.

Points to Consider in Identifying Costs and Benefits for Analysis (OMB Circular A-94, Para 6a1 & 13c1).

Lease-purchase analysis should compare the net present value of the incremental costs related to leasing the asset with the incremental costs related to purchasing (or constructing) and owning the asset. You should consider incremental costs associated with acquisition as well as the

ancillary costs related to acquisition and ownership. Use the following general guidelines as you identify incremental benefits and costs to include in your analysis:

- Analysis should consider costs or benefits associated with one alternative in the evaluation of other alternatives. For example, if the lease payments include maintenance, the purchase alternative should also include the cost of maintenance.
- Analysis should consider costs or benefits that will be different for different alternatives. For example, if different alternatives will use substantially different amounts of electricity, the cost of electricity should be considered.
- Analysis should not consider sunk costs or benefits. Past experience is relevant only in helping to estimate future costs or benefits. For example, if the Government has decided to replace existing equipment, the value of that equipment is not relevant.
- Analysis should not consider costs which will be identical for all alternatives. For example, if the Government has decided to replace existing equipment, the cost of removing that equipment is not relevant because it must be accomplished for all alternatives.

Examples of Lease-Purchase Costs and Benefits Commonly Considered (FAR 7.401 and OMB Circular A-94 Paragraph 13c).

Lease-purchase analysis is one area where you might be required to use net present value analysis. The costs and benefits identified below for lease-purchase analysis demonstrate the type of cash flows that you should consider in a net present value analysis.

- **Net Purchase Price.** Any net present value analysis of a decision to purchase an asset must consider the purchase price. OMB defines the purchase price of the asset as the price a willing buyer could reasonably expect to pay a willing seller in a competitive market to acquire the asset. Normally, lease-purchase decisions do not consider trade-ins of existing equipment. Disposal of existing equipment should be handled following agency property disposal procedures and considered as part of disposal costs and salvage value as presented below.
- **Lease Payments.** Any decision to lease property using net present value analysis must consider the amount and timing of lease payments.
- **Ancillary Services.** If ancillary costs differ between alternatives, they should be considered. (If costs and timing are the same for all alternatives, they need not be considered.) Both OMB Circular A-94 and the FAR provide guidance on the ancillary costs and benefits that you should consider in lease-purchase analysis. The following points combine the recommendations from both sources:
 - All costs associated with acquiring the property and preparing it for use including:
 - Costs;
 - Transportation;
 - Installation;
 - Site preparation;
 - Design; and
 - Management.
 - Repair and improvement costs, including:

- Estimated unplanned service calls; and
 - Improvements projected to be required during the lease period to assure continued operation.
 - Operation and maintenance costs, including:
 - Operating labor and supply requirements; and
 - Routing maintenance.
 - Disposal costs and salvage value, including the:
 - Cost of modifications required to return related equipment to its original configuration;
 - Cost or modifications required to return related facilities to their original configuration; and
 - Equipment value to the Government at the end of the lease period (e.g., salvage value).
-

9.4 - Determining Cash Flow Timing

The timing of cash flows is a vital element of any net present value analysis. This section presents two methods for considering that timing.

- [9.4.1 - Discount Factors for End-of-Year Payment](#)
- [9.4.2 - Discount Factors for Mid-Year Payment](#)

General Equation for Present Value Calculation . You can compute the present value of any cash flow (expenditure/receipt) in the future, by multiplying the amount by the appropriate discount rate:

$$PV = DF(CF)$$

Where:

PV = Present value

DF = Discount factor

CF = Cash flow

Discount Factors . The discount factor that you use in net present value analysis will depend on the discount rate that you use and the timing of the cash flow. In defining the timing of the cash flow, you must identify the year and the timing during the year. There are two commonly used assumptions about when during the year the payment occurs:

- End-of-year payment -- use this assumption when a single payment is made at the end of the year or the beginning of the year.
 - A payment that is due immediately is not discounted.

- A payment that is due at the beginning of Year t is evaluated as a payment due at the end of Year t-1. For example, payments due at the beginning of Year 2 and Year 3 will be treated as if they are due at the end of Year 1 and Year 2.
- Mid-year payment -- use this assumption when a single payment will be made mid-year or payments will be made at regular intervals throughout the year.

Offer-Identified Cash Flows . Solicitations must require all offerors to clearly define the amount and timing of each cash flow (expenditure/receipt) unique to the proposal. The proposal should also include a rationale to support the timing of any cash flow unless the timing is set forth in the contract.

For example: The timing of lease payments does not require any additional support because the timing (e.g., monthly, quarterly, or annually) is defined in the lease agreement. However, the lease agreement may include additional charges (e.g., on-call equipment repair). For such charges, the rationale for both the estimated expenditure and its timing should be clearly defined in the proposal.

Government-Identified Cash Flows . Government technical personnel must identify cash flows related to different proposals that are beyond the control of the offerors.

For example: The amount and timing of expenditures related to Government ownership must also be identified prior to proposal evaluation. Normally, Government personnel will be responsible for preparing these estimates based on available information. However, each offeror may be required to provide information required to develop these estimates (e.g., costs to modify equipment to meet anticipated changes in Government requirements).

9.4.1 - Discount Factors For End-Of-Year Payment

When to Use End-of-Year Discount Factors . Use end-of-year discount factors when payments are due at the end of the year or the beginning of the year. Remember, that a payment due at the beginning of Year 3 is the same as a payment due at the end of Year 2.

End-of-Year Discount Factor Calculation . The discount factor formula for each end-of-year cash flow (payment/ receipt) is written:

$$DF = \frac{1}{(1+i)^t}$$

Where:

DF = End-of-year discount factor

i = Discount rate

t = Number of years until the payment (receipt is due)

For Example : Determine the present value (PV) of a payment of \$1,000 due at the end of 1 year using the nominal discount rate for three years or less, 5.6 percent.

Discount Factor Calculation:

$$\begin{aligned}
 DF &= \frac{1}{(1 + i)^t} \\
 &= \frac{1}{(1 + .056)^1} \\
 &= \frac{1}{1.056^1} \\
 &= \frac{1}{1.056} \\
 &= .9470
 \end{aligned}$$

Present Value Calculation:

$$\begin{aligned}
 PV &= DF (CF) \\
 &= .9470 (\$1,000) \\
 &= \$947 \text{ (rounded to the nearest dollar)}
 \end{aligned}$$

Sum Factors for Repetitive End-of-Year Cash Flows . When there is a repetitive cash flow such as a lease payment, you can use a sum factor to speed the calculation process.

$$PV = SF (CF)$$

Where:

PV = Present value

SF = End-of-year sum factor

CF = Cash flow

For example : Determine the present value of a series of three payments of \$1,000 each due at the end of each of the next three years, when the discount rate is 5.6 percent.

Year	Payment	Formula	Calculation	Discount Factor (DF)	Present Value (PV)
1	\$1,000	$1/(1.056)^1$	$1/1.0560$.9470 ^a	\$ 947 ^b
2	1,000	$1/(1.056)^2$	$1/1.1151$.8968	\$ 897
3	1,000	$1/(1.056)^3$	$1/1.1776$.8492	\$ 849
Total				2.6930	\$2,693

^a Factors are rounded to the four decimal places.

^b Amounts are rounded to the nearest dollar.

The present value of a series of three \$1,000 end-of-year payments is \$2,693 when the discount rate is 5.6 percent. The sum of the three factors is 2.6930. Using the sum factor and the equation above:

$$\begin{aligned}PV &= SF(CF) \\ &= 2.6930 (\$1000) \\ &= \$2,693 \text{ present value (rounded to the nearest dollar)}\end{aligned}$$

Note: The answer calculated using the sum factor is the same as the answer calculated using individual discount factors. However, answers may vary slightly because of differences in rounding.

End-of-Year Nominal Discount Tables . [Appendix A-1, Discount Factors -- Nominal Rates](#) , End-of-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

End-of-Year Real Discount Tables . [Appendix A-3, Discount Factors -- Real Rates](#) , End-of-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

9.4.2 - Discount Factors For Mid-Year Payment

When to Use Mid-Year Discount Factors . Use mid-year discount factors when a single payment will be made mid-year or payments will be made at regular intervals (e.g., monthly or quarterly) throughout the year.

Mid-Year Discount Factor Calculation. The discount factor formula for mid-year cash flow (payment/receipt) is written:

$$MYDF = \frac{1}{(1 + i)^{(t-.5)}}$$

Where:

MYDF = Mid-year discount factor

i = Discount rate

t = Number of years until the payment (receipt) is due

For example : Determine the present value of a series of 12 monthly payments of \$1,000 each due at the beginning of each month for 1 year. The total amount for the year is \$12,000. These payments are spaced evenly over the year; hence the use of a MYDF would be appropriate.

Discount Factor Calculation:

$$\begin{aligned} \text{MYDF} &= \frac{1}{(1+i)^{(t-.5)}} \\ &= \frac{1}{(1.056)^{(1-.5)}} \\ &= \frac{1}{1.056^{\.5}} \\ &= \frac{1}{1.0276} \\ &= .9731 \end{aligned}$$

Present Value Calculation:

$$\begin{aligned} \text{PV} &= \text{MYDF} (\text{CF}) \\ &= .9731 (\$12,000) \\ &= \$11,677 \text{ (rounded to the nearest dollar)} \end{aligned}$$

Sum Factors for Repetitive Mid-Year Cash Flows . When there is a repetitive cash flow such as a lease payment, you can use sum factors to speed the calculation process.

$$\text{PV} = \text{MYSF} (\text{CF})$$

Where:

PV = Present value

MYSF = Mid-year sum factor

CF = Cash flow

For example : Determine the present value of a series of 36 monthly payments of \$1,000 each due at the beginning of each month for the next three years; that is, \$12,000 per year for three years. These payments are spaced evenly over the year; hence the use of a MYDF would be appropriate.

Year	Payment	Formula	Calculation	Discount Factor (MYDF)	Present Value (PV)
1	\$12,000	$1/(1.056)^{.5}$	$1/1.0276$.9731a	\$11,677b
2	\$12,000	$1/(1.056)^{1.5}$	$1/1.0852$.9215	\$11,058
3	\$12,000	$1/(1.056)^{2.5}$	$1/1.1459$.8727	\$10,472
Total				2.7673	\$33,207

^a Factors are rounded to the four decimal places.

^b Amounts are rounded to the nearest dollar.

The present value of a series of three \$12,000 mid-year payments is \$33,207, when the discount rate is 5.6 percent. The sum of the three mid-year discount factors is 2.7673. Using the sum factor and the equation above:

$$\begin{aligned} PV &= MYSF (CF) \\ &= 2.7673 (\$12,000) \\ &= \$33,208 \text{ present value (rounded to the nearest dollar).} \end{aligned}$$

Note : The answer calculated using the sum factor is slightly higher than the one calculated using individual discount factors, because of rounding differences.

Mid-Year Nominal Discount Tables . [Appendix A-2, Discount Factors -- Nominal Rates](#) , Mid-Year Payments, contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

Mid-Year Real Discount Tables . [Appendix A-4, Discount Factors -- Real Rates, Mid-Year Payments](#) , contains factors for the 3-year, 5-year, 7-year, 10-year, and 30-year discount rates.

9.5 - Calculating Net Present Value And Selecting The Best Alternative

Net Present Value Analysis . Remember from the Chapter Introduction that you should use the following 5-step process in net present value analysis:

Step 1. Select the discount rate.

Step 2. Identify the costs/benefits to be considered in analysis.

Step 3. Establish the timing of the costs/benefits.

Step 4. Calculate net present value of each alternative.

Step 5. Select the offer with the best net present value.

This section will demonstrate the use of that 5-step process in two lease-purchase decision examples using nominal discount rates. You should follow the same steps for any net present value analysis whether you are using nominal discount rates or real discount rates.

Lease-Purchase Decision Example 1 . Assume that you want to determine which of the following proposals will result in the lowest total cost of acquisition?

Offeror A : Proposes to lease the asset for 3 years. The annual lease payments are \$10,000 per year. The first payment will be due at the beginning of the lease; the remaining two payments are due at the beginning of Years 2 and 3.

Offeror B : Proposes to sell the asset for \$29,000. It has a 3-year useful life. Salvage value at the end of the 3-year period, will be \$2,000.

Step 1. Select the discount rate. The term of the lease analysis is three years, so we will use the nominal discount rate for three years, 5.4 percent.

Steps 2 and 3. Identify and establish the timing of the costs/benefits to be considered in analysis. The expenditures and receipts associated with the two offers and their timing are delineated in the table below: (Parentheses indicate a cash outflow.)

Offer-Related Expenditures/Receipts		
t	Offer A	Offer B
0	\$10,000	\$29,000
1	\$10,000	-0-
2	\$10,000	-0-
3	-0-	(\$2,000)

Step 4. Calculate net present value. The tables below summarize the net present value calculations applied to each alternative.

Net present value of Offer A			
t	Cash Flow	DF	PV
0	\$10,000	1.0000	\$10,000
1	\$10,000	0.9470	\$9,470
2	\$10,000	0.8968	\$8,968
Net Present Value			\$28,438

Note the following points in the net present value calculations above:

- - Payments due now are not discounted.
 - Payments due at the beginning of Years 2 and 3 are treated as if they are due at the end of Years 1 and 2.
 - You could have calculated the net present value of Offer A using the Sum of Discount Factors (Appendix A-1) for the payments due at the beginning of Years

2 and 3. Remember that payments due now are not discounted and payments due at the beginning of Years 2 and 3 are treated as if they are due at the end of Years 1 and 2. The calculations would be:

$$\begin{aligned} \text{NPV} &= -\$10,000 + 1.8438(-\$10,000) \\ &= -\$10,000 - \$18,438 \\ &= -\$28,438 \end{aligned}$$

Net present value of Offer B			
t	Cash Flow	DF	PV
0	\$29,000	1.0000	\$29,000
3	(\$2,000)	0.8492	(\$ 1,698)
Net Present Value			\$27,302

Note the following points in the net present value calculations above:

- - Offer B salvage value is treated as a cash inflow at the end of Year 3.
 - Payments due now are not discounted.

Step 5. Select the offer with the best net present value. In this example, we would select Offer B, the offer with the lowest net present value.

Lease-Purchase Decision Example 2 . Assume that we want to determine which of the following proposals will result in the lowest acquisition cost?

Offeror A -Proposes to lease the asset for 3 years. The monthly lease payments are \$1,500; that is, the total amount for each year is \$18,000. These payments are spaced evenly over the year, so the use of a MYDF would be appropriate.

Offeror B -Proposes to sell the asset for \$56,000. It has a 3-year useful life. At the end of the 3-year period it will have a \$3,000 salvage value.

Step 1. Select the discount rate. The term of the analysis is three years, so we will use the nominal discount rate for three years, 5.6 percent.

Steps 2 and 3. Identify and establish the timing of the costs/benefits to be considered in analysis. The expenditures and receipts associated with the two offers and their timing are delineated in the table below:

Offer Expenditures/Receipts

t	Offer A	Offer B
0	-0-	\$56,000
1	\$18,000	-0-
2	\$18,000	-0-
3	\$18,000	(\$3,000)

Step 4. Calculate net present value. The tables below summarize the net present value calculations applied to each alternative.

Net present value of Offer A			
t	Cash Flow	DF	PV
1	\$18,000	0.9731	\$17,516
2	\$18,000	0.9215	\$16,587
3	\$18,000	0.8727	\$15,709
Net Present Value			\$49,812

NOTE the following points in the net present value calculations above:

- - Offeror A payments are due monthly, so we used the nominal rate, mid-year factors from [Appendix A-2](#) .
 - You could also calculate the net present value of Offer A using the Sum of Discount Factors in [Appendix A-2](#) . That calculation would produce a slightly different answer due to rounding differences.

Net present value of Offer B			
t	Cash Flow	DF	PV
0	\$56,000	1.0000	\$56,000
3	(\$3,000)	0.8492	(\$2,548)
Net Present Value			\$53,452

Note the following points in the net present value calculations above:

- - Offer B salvage value is treated as a cash inflow at the end of Year 3.
 - Payments due now are not discounted.

Step 5. Select the offer with the best net present value. In this example, we would select Offer A, the offer with the lowest net present value.

9.6 - Identifying Issues And Concerns

Questions to Consider in Analysis . As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use net present value analysis.

- *Is net present value analysis used when appropriate?*

Net present value analysis should be used in any analysis supporting Government decisions to initiate, renew, or expand programs or projects which would result in a series of measurable benefits or costs extending for three or more years into the future.

- *Are the dollar estimates for expenditures and receipts reasonable?*

The base for all present value calculations is estimated future cash flows. The rationale for those estimates must be documented and supported just like any cost estimate. This includes estimates of costs that will be included in the contract or lease agreement and estimates of other cash flows that are not included. All may be used in present value calculations.

- *Are the times projected for expenditures and receipts reasonable?*

Discount factors depend on the discount rate and the timing of the cash flow. The timing of any cash flow not documented in the contract or lease agreement must be clearly supported. The offeror is responsible for estimating and defending cash flow estimates included in the proposal. Government technical personnel have that responsibility for estimated costs related to item ownership.

- *Are the proper discount rates used in the net present value calculations?*

Unless precluded by agency policy, discount rates should be taken from Appendix C of OMB Circular A-94. If they are not, the rationale must be documented.

- The rate selected should be based on the number of time periods included in the analysis. If the period of the analysis does not match any of the discount rate periods delineated in OMB Circular A-94, linear interpolation should be used to estimate a rate for that period of time.
 - - Nominal discount rates should be used for any analysis not based on constant year dollars. Real discount rates should be used for any analysis that is based on constant year dollars.
- *Are the proper discount factors used in analysis?*

The discount factor should be calculated considering the timing of the cash flow.

- - End-of-year discount factors (Appendices [A-1](#) and [A-3](#)) should be used for cash flows at the beginning or end of the year.
 - Mid-year discount factors (Appendices [A-2](#) and [A-4](#)) should be used for cash flows in the middle of the year or regularly throughout the year (e.g., monthly or quarterly).
- *Are discount factors properly calculated from the discount rate?*

End-of-year or mid-year discount rates should be calculated following the procedures delineated in [Section 9.4](#).

- - *Have all cash flows been considered?*

Net present value analysis must consider all relevant cash flows throughout the decision life cycle.