Two-tailed hypothesis test example

Problem: A premium golf ball production line must produce all of its balls to 1.615 ounces in order to get the top rating (and therefore the top dollar). Samples are drawn hourly and checked. If the production line gets out of sync with a statistical significance of more than 1%, it must be shut down and repaired. This hour's sample of 18 balls has a mean of 1.611 ounces and a standard deviation of 0.065 ounces. Do you shut down the line?

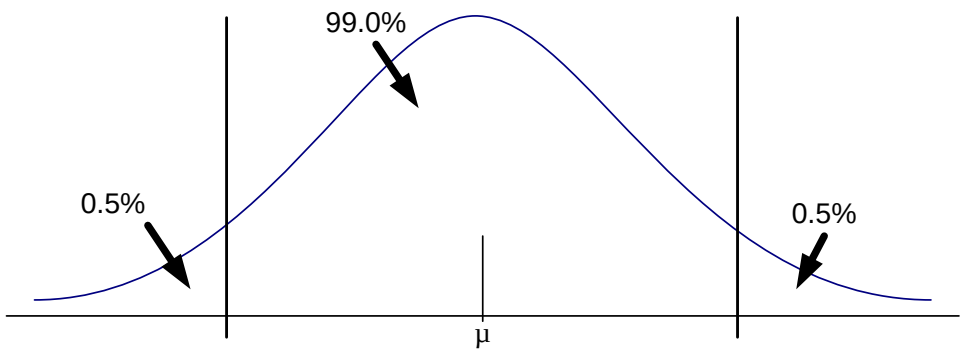
Answer:

Step 1. Determine the null hypothesis.

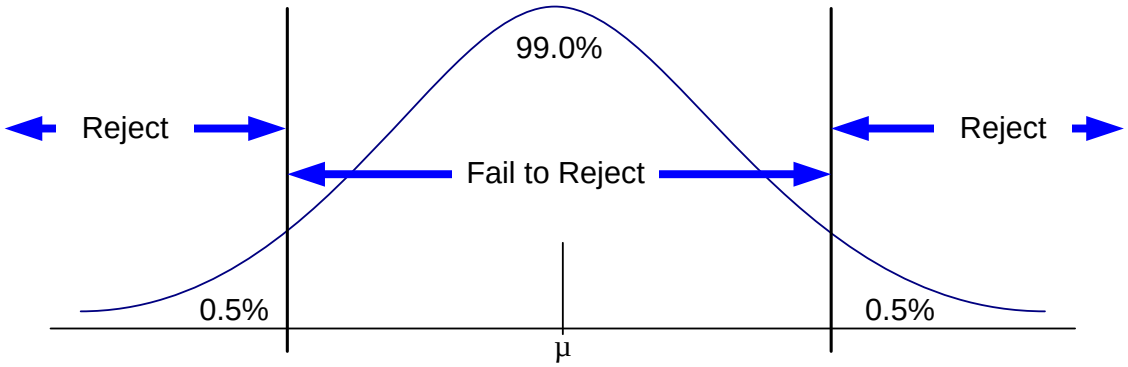
H_0: The population mean (μ) = 1.615

H_A: The population mean ≠ 1.615 (hence a 2-tailed test)

Step 2. Draw the t-test diagram



Assign the "Fail to reject" to the appropriate region.



Step 3. Determine and Label t_p.

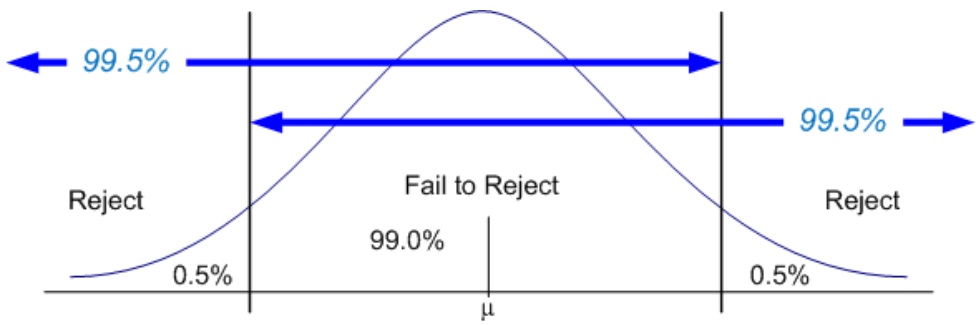
Since our table (page 5-20 in the text) is a one-tailed table and we are doing a two-tailed test, we have to divide the level of significance in half.

$$\alpha = 0.010$$

$$\frac{\alpha}{2} = 0.005$$

$$p = 1 - \frac{\alpha}{2}$$

$$p = 0.995 = 99.5\%$$



Note that the Bold Blue lines are not part of the normal drawing. They are added here to show that the appropriate value to look up on a one-tailed table when doing a 2-tailed test.

The degrees of freedom are sample size minus one.

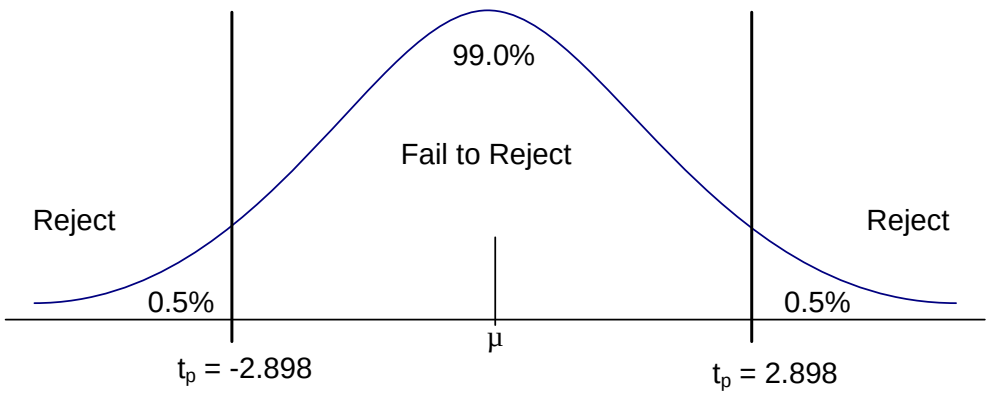
$$df = 18 - 1$$

$$df = 17$$

From the tables on page 5-20 of the text

df	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	.325	.727	1.376	3.078	6.314	12.706	31.821	63.656
2	.289	.617	1.061	1.886	2.920	4.303	6.965	9.925
3	.277	.584	.978	1.638	2.353	3.182	4.541	5.841
4	.271	.569	.941	1.533	2.132	2.776	3.747	4.604
5	.267	.559	.920	1.476	2.015	2.571	3.365	4.032
6	.265	.553	.906	1.440	1.943	2.447	3.143	3.707
7	.263	.549	.896	1.415	1.895	2.365	2.998	3.499
8	.262	.546	.889	1.397	1.860	2.306	2.896	3.355
9	.261	.543	.883	1.383	1.833	2.262	2.821	3.250
10	.260	.542	.879	1.372	1.812	2.228	2.764	3.169
11	.260	.540	.876	1.363	1.796	2.201	2.718	3.106
12	.259	.539	.873	1.356	1.782	2.179	2.681	3.055
13	.259	.538	.870	1.350	1.771	2.160	2.650	3.012
14	.258	.537	.868	1.345	1.761	2.145	2.624	2.977
15	.258	.536	.866	1.341	1.753	2.131	2.602	2.947
16	.258	.535	.865	1.337	1.746	2.120	2.583	2.921
17	.257	.534	.863	1.333	1.740	2.110	2.567	2.898
18	.257	.534	.862	1.330	1.734	2.101	2.552	2.878

t_p= 2.898. Add this information to the drawing.



Step 4. Calculate t_{calc} or also known as t_c

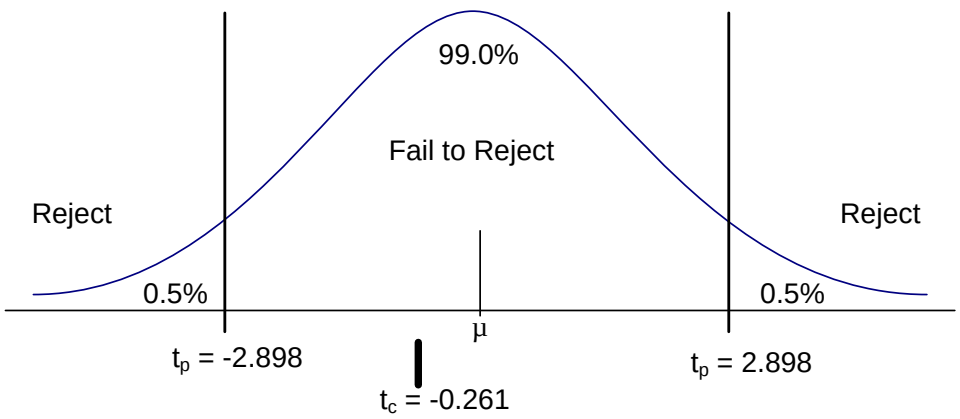
$$t_c = \frac{\bar{Y} - \mu}{s/\sqrt{n}}$$

$$t_c = \frac{1.611 - 1.615}{0.065/\sqrt{18}}$$

$$t_c = \frac{-0.004}{0.0153}$$

$$t_c = -0.261$$

Add to the graph



Step 5. Determine your conclusions.

Since t_c is clearly in the "Fail to Reject" region, then we must "fail to reject the null hypothesis". The null hypothesis was "H_0: The population mean = 1.615". Failing to reject that means that the sample was within the bounds of what we would find acceptable if the population mean was 1.615 ounces.

Ergo, we would NOT stop the production line.