One-tailed hypothesis test example

Problem: An alternator manufacturer must produce its alternators so that they are 95% confident that it runs at less than 71.1^{o} C under stress test in order to meet the production requirements for sale to the US government. The stress test is performed on random samples drawn from the production line on a daily basis. Today's sample of 7 alternators has a mean of 71.3^{o} C and a standard deviation of 0.214^{o}. Is there a production quality issue?

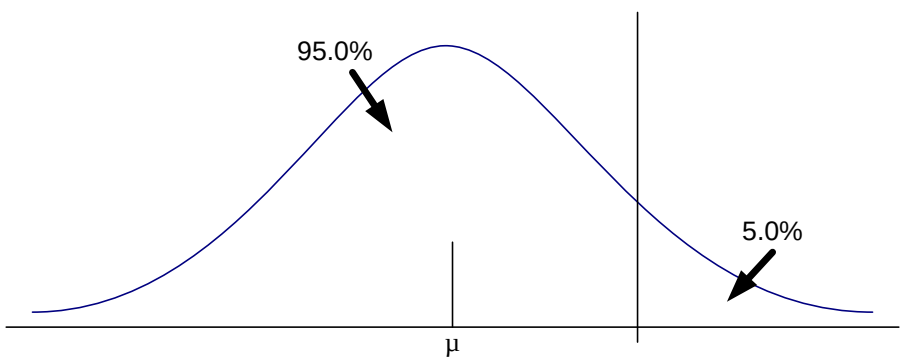
Answer:

Step 1. Determine the null hypothesis.

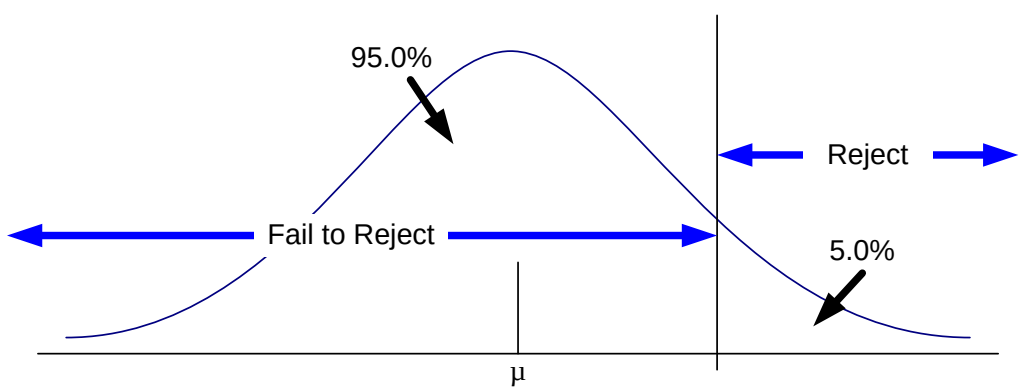
H_0: The population mean (μ_0) ≤ 71.1

H_A: The population mean > 71.1

Step 2. Draw the t-test diagram



Assign the "Fail to reject" to the appropriate region.



Step 3. Determine and Label t_p.

Since our table (page 5-20 in the text) is a one-tailed table and we are doing a one-tailed test, we do nothing special with the level of significance.

$$p = 95.0\% = 0.95$$

If we were interested in finding the level of significance, it would just be $\alpha = 1 - p$ which in this case is just $\alpha = 0.05$.

The degrees of freedom are sample size minus one.

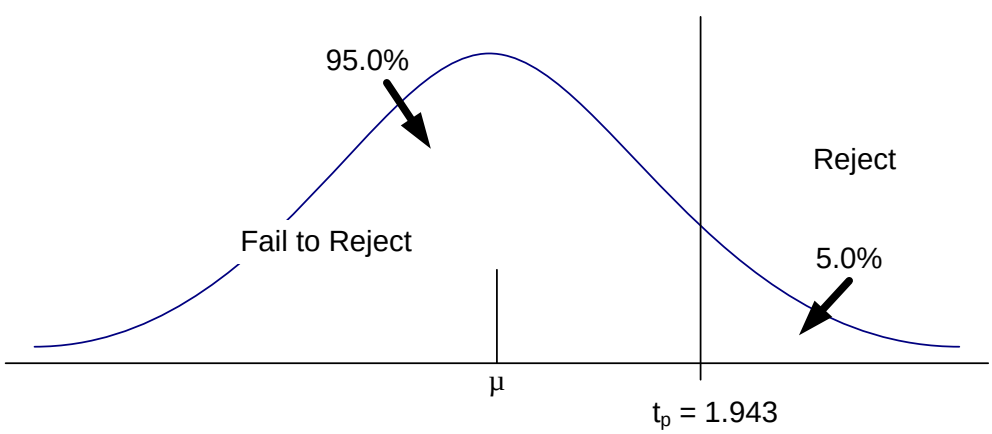
$$df = 7 - 1$$

$$df = 6$$

From the tables on page 5-20 of the text,

df	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.90}$	$t_{.95}$	$t_{.975}$
1	.325	.727	1.376	3.078	6.314	12.706
2	.289	.617	1.061	1.886	2.920	4.303
3	.277	.584	.978	1.638	2.353	3.182
4	.271	.569	.941	1.533	2.132	2.776
5	.267	.559	.920	1.476	2.015	2.571
6	.265	.553	.906	1.440	1.943	2.447
7	.263	.549	.896	1.415	1.895	2.365

t_p= 1.943. Add this information to the drawing.



Step 4. Calculate t_{calc} or also known as t_c

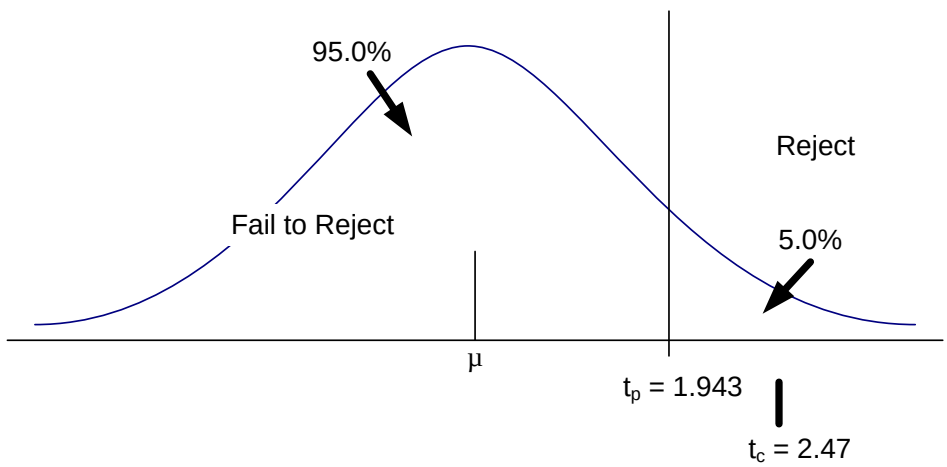
$$t_c = \frac{\bar{Y} - \mu}{s/\sqrt{n}}$$

$$t_c = \frac{71.3 - 71.1}{0.214/\sqrt{7}}$$

$$t_c = \frac{0.2}{0.0809}$$

$$t_c = 2.47$$

Add to the graph



Step 5. Determine your conclusions.

Since t_c is clearly in the "Reject" region, then we must "reject the null hypothesis". The null hypothesis was "H_0: The population mean is less than or equal to 71.1° C". Rejecting that means that the sample was NOT within the bounds of what we would find acceptable if the population mean were truly at 71.1° C.

Ergo, we would claim there is a problem with the production of alternators.